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# Optimal Performance of Horizontal Axis Wind Turbine for Low Wind Speed Regime

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***Abstract***

*The development of performance prediction is one of the most important aspects of the design of wind turbines. In this paper, a developed methodology is used to predict the optimal performance of the horizontal axis wind turbine in terms of the most critical parameters such as tip speed ratio, pitch angle, blade number and wind speed. Interesting generalized performance maps were conducted. Results show that low pitch is recommended for low wind speed regime. A range of (5 to 11) of tip speed ratio is found an optimum within the constraints considered. The interplay of cut in speed with the remaining parameters is also studied and their effect on power and torque are explored. Several results were presented for a three bladed wind turbine is it is preferred by many manufacturers and researchers.*

***Keywords***: *Wind turbine Pitch angle, Number of blades, Bitz limit, H.A.W.T, Power Coefficient, TSR, Low wind speed.*

## 1. Introduction

The efficiency of any wind energy conversion system is described in terms of its power coefficient, Cp **[1]**. Design parameter choice is critical for optimizing wind turbine performance. For any fixed diameter there are various parameters influencing energy production: rotor rotation velocity, blade number, airfoil chord distribution and longitudinal blade twist. **[2]**. Recent researches focuses on the optimum operation parameters of wind turbine. **Isaac et. al.[3]** developed different statistical models of wind to estimate power coefficient of wind turbine . They calculated two parameters Weibull distribution function at three different locations. They attributed that the most influential factor for any wind energy conversion system is the wind velocity on the available site. **Someers et. al. [4]** designed a series of airfoil using BEM theory for large wind turbines using XFOIL software. They designed airfoils for maximum lift to maintain high aerodynamic efficiency when operating in varying wind conditions. It was concluded that optimum airfoil shape of blade dictates the aerodynamic performance of wind turbine and hence annual power production of a turbine. **Doucette et.al. [5]** conducted a numerical study to examine the impact of rotor solidity and lead to increase power coefficients at lower tip speed ratios. **Sinopoli et.al.[6]** modeled wind turbine rotor using statistical response surface methodology and observed that for every value of pitch angle there is a unique value of TSR that optimize the power coefficient of wind turbine.

The wind power varies so greatly (with the cube of wind speed), the turbine must be able to generate power in light winds and withstand the loads in much stronger winds. Therefore, above the optimum wind speed, the blades are typically pitched either into the wind or away from the wind to reduce the generated power and regulate the loads.

This paper focus on the optimum performance parameters that affect the power coefficient, the thrust coefficient, turbine output power and thrust. These parameters are the free stream wind velocity, axial induction factor, and pitch angle, number of blades, angle of attack and cut in wind velocity. The optimum region of these parameters are obtained by using MATLAB simulation program.

## 2-Methodology

The major aspect of wind turbine performance like power output and loads are determined by the aerodynamic forces generated by the wind. These can only be understood with a deep comprehension of the aerodynamics of steady state operation. The BEM theory combines two methods to analyze the aerodynamic performance of wind turbine. These are momentum theory and blade - element theory which are used to outline the governing equations for the aerodynamic design and power prediction of a wind turbine rotor. Momentum theory examines the momentum balance on rotating annular stream tube passing through a turbine and blade-element theory examines the forces generated by aerofoil lift and drag coefficients at various sections along the blade **[7].**

*2-1- The Actuator Disc Theory and the BETZ Limit*

The power produced by wind turbine can be obtained by multiplying the power available in wind by power coefficient, the power available in wind depends on air density, rotor swept area and cubic free stream wind velocity, thus the turbine power can be expressed as:

$$P = \frac{1}{2}\rho A U_\infty^3 C_p \qquad (1)$$

The axial induction factor a indicates the degree with which the wind velocity at the upstream of rotor slowed down by the turbine. Thus:

$$a = \frac{U_\infty - U_R}{U_\infty} \qquad (2)$$

Wind velocity before and after the actuator disk equal to wind velocity at rotor plane. Thus:

$$U_2 = U_3 = U_R \qquad (3)$$

The power of turbine can be expressed in terms of axial induction factor as follow:

$$P = 2\rho A a(1-a)^2 U_\infty^3 \qquad (4)$$

The power coefficient can be expressed in terms of axial induction factor as follow:

$$C_P = 4a(1-a)^2 \qquad (5)$$

The power coefficient can be presented as a function of free stream wind velocity and velocity at rotor plane by substituting equation (2) into equation (5).

$$Cp = 4((\frac{U_\infty - U_R}{U_\infty}) - 2(\frac{U_\infty - U_R}{U_\infty})^2 + (\frac{U_\infty - U_R}{U_\infty})^3) \qquad (6)$$

The amount of power produced by wind turbine can be presented by substituting equation (6) into equation (1)

$$P = 2\rho A(U_\infty - U_R)[U_\infty^2 - 2U_\infty(U_\infty - U_R) + (U_\infty - U_R)^2] \qquad (7)$$

The torque developed by turbine shaft can be obtained by multiplying theoretical torque by thrust coefficient, theoretical torque depends on air density, rotor swept area, rotor radius and squared free stream wind velocity, thus the turbine torque can be expressed as:

$$T = \frac{1}{2}\rho A U_\infty^2 R\, C_T \qquad (8)$$

It also can be presented in terms of axial induction factor as follow:

$$T = 2\rho A a(1-a)U_\infty^2 \qquad (9)$$

The thrust coefficient in terms of axial induction factor can be presented as follow:

$$CT = 4a\,(1-a) \qquad (10)$$

The thrust coefficient can be presented as a function of free stream wind velocity and velocity at rotor plane by substituting of equation (2) into equation (10).

$$C_T = 4((\frac{U_\infty - U_R}{U_\infty}) - \left(\frac{U_\infty - U_R}{U_\infty}\right)^2) \qquad (11)$$

The torque of wind turbine can be re-written in terms of free stream wind velocity and velocity at rotor plane by substituting equation (11) into equation (8).

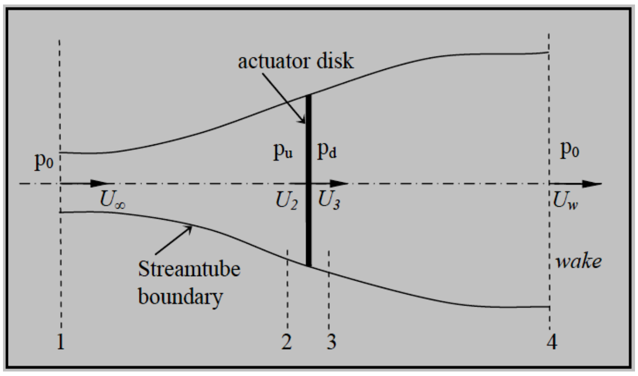
$$T = 2\rho A U_R(U_\infty - U_R) \qquad (12)$$

Tip speed ratio can be expressed as the ratio of power coefficient to thrust coefficient. Thus:

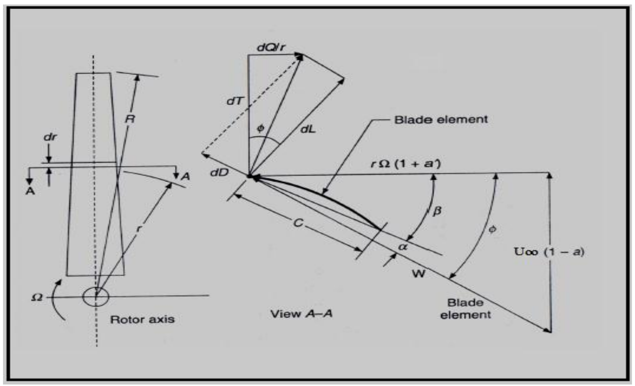
$$\lambda = \frac{C_p}{C_T} \qquad (13)$$

It can be expressed in another formula as follow:

$$\lambda = (1 - \left(\frac{U_\infty - U_R}{U_\infty}\right)) \qquad (14)$$




(a)




(b)

**Figure (1)**: (a) velocity and force components on a blade element**[8]** , (b) Idealized flow through a wind turbine represented by a non-rotating, actuator disk

*2-2- Power coefficient in terms of pitch angle and TSR*

The turbine is characterized by its power coefficient Cp which is function of the tip speed ration and the pitch angle of specific wind turbine blades. For the used turbine, this coefficient is given by the following mathematical approximation **[9]:**

$$\mathrm{Cp} = 0.5176\left(\frac{116}{\lambda_i} - 0.4\beta - 5\right)e^{\frac{-21}{\lambda_i}} + 0.0068\lambda \quad (15)$$

Where

$$\frac{1}{\lambda_i} = \frac{1}{\lambda+0.08\beta} - \frac{0.035}{\beta^3+1} \quad (16)$$

In order to express the power coefficient in another mathematical formula, substitute equation (16)
Into equation (15) as follow:

$$C_P = 0.5176\left(\frac{116}{\left(\frac{1}{\frac{1}{\lambda+0.08\beta}-\frac{0.035}{\beta^3+1}}\right)} - 0.4\beta - 5\right)e^{-21/\left(\frac{1}{\frac{1}{\lambda+0.08\beta}-\frac{0.035}{\beta^3+1}}\right)} + 0.0068\lambda \quad (17)$$

Where :

$$\lambda = \frac{\omega R}{U_\infty} \quad (18)$$

With the function defined in equation (17), it is possible to evaluate of Cp, response, at different values of tip speed ratio and pitch angle **[9].**

*2-3-Power coefficient and TSR in terms of number of blades*

The power coefficient depends on the design of the turbine and the wind speed. According to Betz law the coefficient has a maximum of 16/27. Thus the maximum mechanical energy that can be extracted from the wind is 16/27 times the energy in wind. Because many design variables of a turbine influence the power coefficient it is very useful to make a quick approximation of the power coefficient with a particular turbine design. An empirical equation that gives an approximation of the maximum power coefficient is the equation of Wilson **[10]:**

$$C_{p,max} = \left[\frac{\lambda B^{0.67}}{1.48+(B^{0.67}-0.04)\lambda+0.0025\lambda^2} - \frac{1.92\lambda^2 B}{1+2\lambda B} D/L\right] \quad (19)$$

The tip speed ratio is an extremely important factor in wind turbine design, which is defined as the ratio of the tangential speed at the blade tip to the actual wind speed, i.e.:

$$\lambda = \frac{(1+r)\omega}{U_\infty} \quad (20)$$

If the blade angular speed ω is too small, most of the wind may pass undisturbed though the blade swept area making little useful work on the blades. On the contrary, if ω is too large, the fast rotating blades may block the wind flow reducing the power extraction. Therefore, there exists an optimal angular speed at which the maximum power extraction is achieved. For a wind turbine with B blades, the optimal angular speed can be approximately determined as **[11]:**

$$\omega_{opt} = \frac{2\pi U_\infty}{BL_1} \quad (21)$$

Substituting equation (21) into (20), the optimal tip speed ratio becomes*:*

$$\lambda_{opt} \approx \frac{2\pi}{B}\left(\frac{1+r}{L_1}\right) \quad (22)$$

Empirically, the ratio ( l + r )/ $L_1$ is equal to about 2.

Thus:

$$\lambda_{opt} = \frac{4\pi}{B} \quad (23)$$

Substituting equation (23) into equation (19)

$$C_{p,max} = \left[1 + \frac{12.56B^{2.34}}{1.48B^{2.67}-0.502B^{1.67}+0.394B^{0.67}} - \frac{11.6}{B}D/L\right] \quad (24)$$

For such a linear turbine (simplified case) the power coefficient can be calculated with:

$$C_P = C_{P,max} * 6.75 * \frac{U_{in}^3}{U_\infty^3}\left(\frac{U_\infty}{U_{in}} - 1\right) \quad (25)$$

Cut in wind speed is the minimum wind speed at which the machine will deliver useful power. It's typical value will be approximately $0.6V_m$ **[12].**

**3-Performance Prediction of Horizontal Axis Wind Turbine**

There are many strategies to estimate the optimal performance parameters, one of these strategies, using computer program to execute the functions that describes the main performance parameters. In this paper the input data into computer program are radius of turbine rotor, air density, axial induction factor, pitch angle rotational speed of turbine, number of blades, cut in and free stream wind velocity. The flow chart shown below constructed for estimation process of main performance parameters, it shows that the power and thrust coefficients in terms of BITZ limit depends on the axial induction factor and then used to estimate tip speed ratio, in terms of pitch angle, the thrust and power coefficient depends on pitch angle and tip speed ratio, in

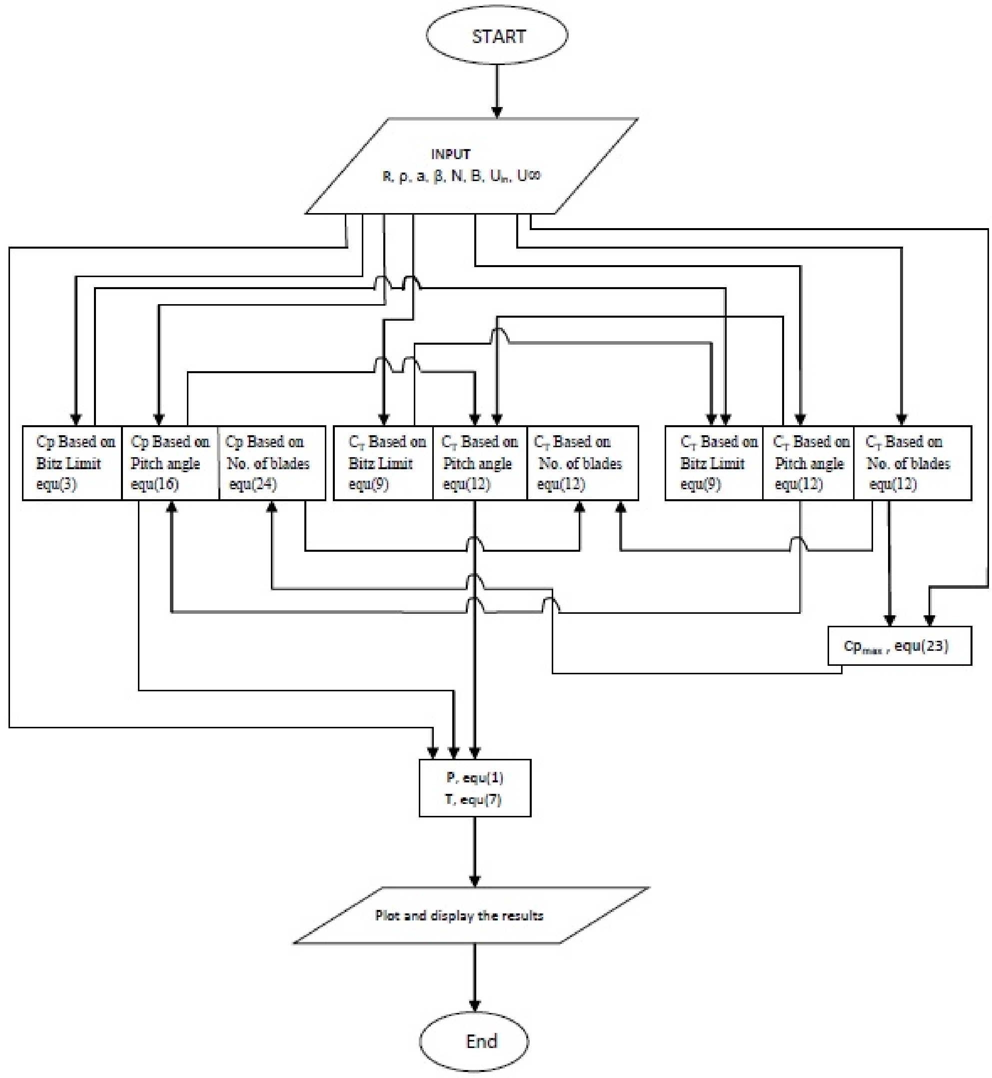



**Figure (2)** The flow chart of the computer program

terms of number of blades the power and thrust coefficient depends on number of blades and tip speed ratio and cut in wind velocity.

**4- Results and discussions**

A comprehensive look at a turbine blade configuration has shown that an efficient blade shape is achieved by aerodynamic calculations based on chosen parameters and the performance of the selected aerofoil. A best representation of power coefficient, TSR and thrust coefficient with axial induction is shown in figure (3). Maximum power coefficient refers to trading of (1/3) while maximum thrust coefficient excited at axial induction of (0.5) these values can be obtained by Bitz turbine .TSR varies inversely with axial induction.

Power produced by wind turbine versus axial induction for a range of velocities and radii are presented in figures (4) and (5) respectively. Important generalized gproductivity and turbine capability of extraction is presented for specified range of velocities and different pitch angles. The impact of blade pitch angle is a critical parameter for the aerodynamic optimization of untwisted blades.

Negative pitch angle of ($5^o$) is suited for lowest wind speed regime. It enhances the capability of the wind turbines, increasing pitch angle will leads to higher power at high wind speed. This status can be synchronized through the control process.

Increasing TSR (i.e. rotational speed or rotor diameter) changes the trend of the turbine performance. Negative pitch angle possess high power coefficient at high wind speed. A range of TSR of the order (0.6 – 11) may give best performance of wind turbine similar behavior of the torque to appeared as the torque is function of power for specified rotational speed. This is shown in figure (10).

Researchers try to overcome the difficulty of starting of wind turbine via an optimization of the decisive parameters, starting at low wind speed, leading to better performance in low wind speed regime. Large scale turbine start at a relatively high cut in speed, but they reveal high power coefficient for a wider range of wind speeds. Turbine with low cut in speeds easily passed the torque possessed. High wind speeds and rotor spans lead to higher torque and wind turbines got starting at high cut in speed.

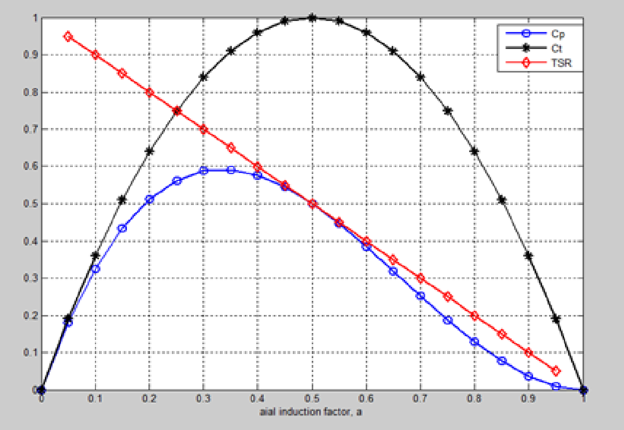



**Figure (3)** :the relationship between ( Cp , Ct and λ) and axial induction factor(a)

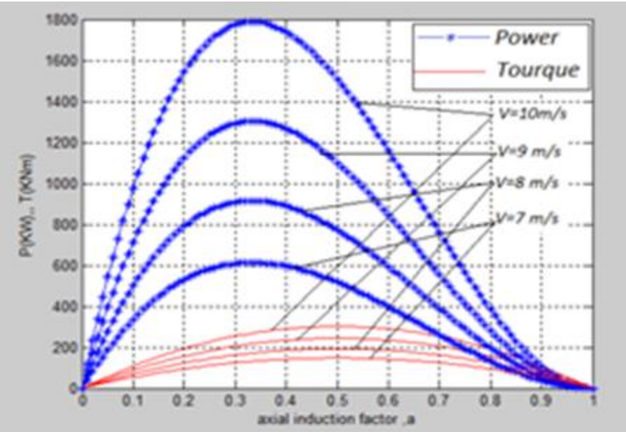
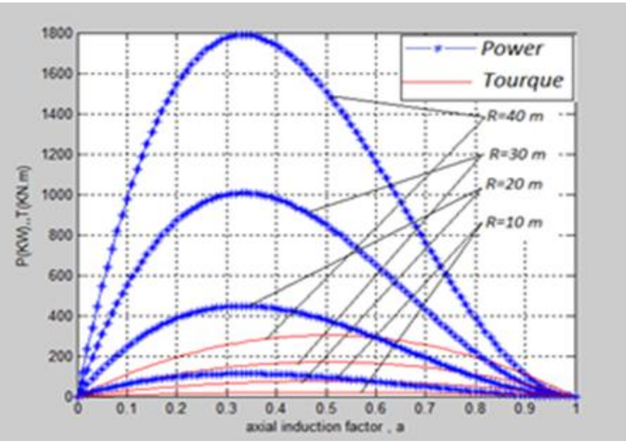



**Figure (4):** the relationship between P(KW) and axial induction factor (a) for different turbine rotor radii.




**Figure(5):** Relationship P(KW) and axial Induction factor (a) for different turbine radii

For reasons of efficiency, control, noise and aesthetics the modern wind turbine market is dominated by the horizontally mounted three blade design. Thus the three blade rotor is the most important and most visible part of the wind turbine.

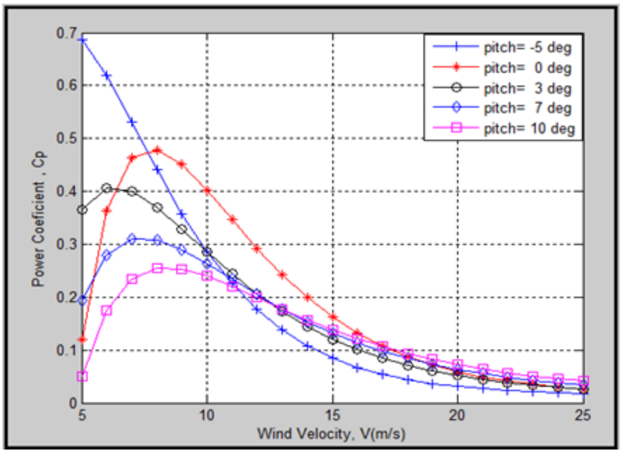



**Figure (6)** : Relationship between Power coefficient and wind Velocity for different pitch angles

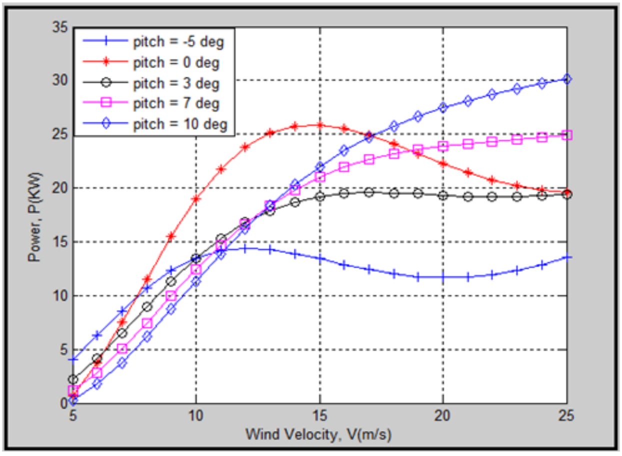



**Figure (7)**: Relationship between P(KW) and wind velocity for different pitch angles

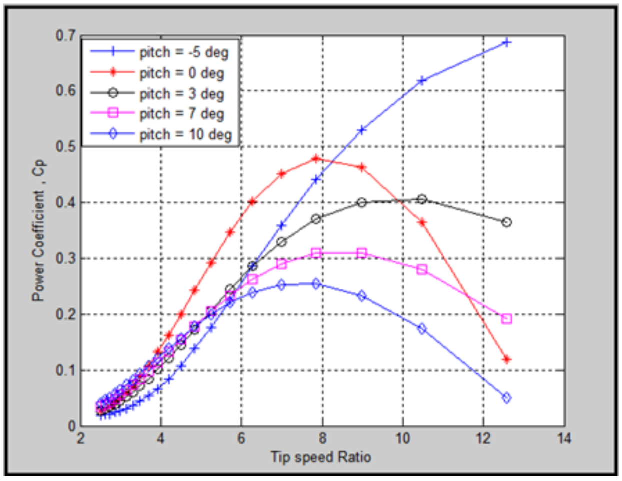



**Figure (8):** the relationship between thrust coefficients and wind velocity for different pitch angles.

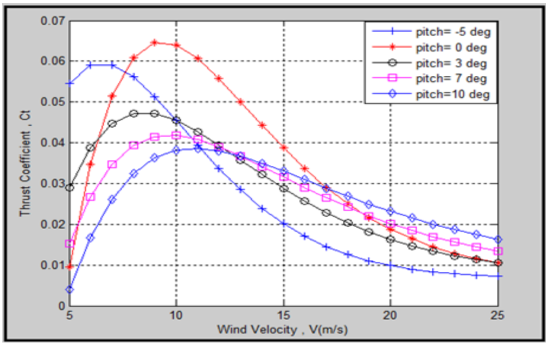



**Figure (9):** the relationship between Tip Power coefficients and wind velocity for different pitch angles.

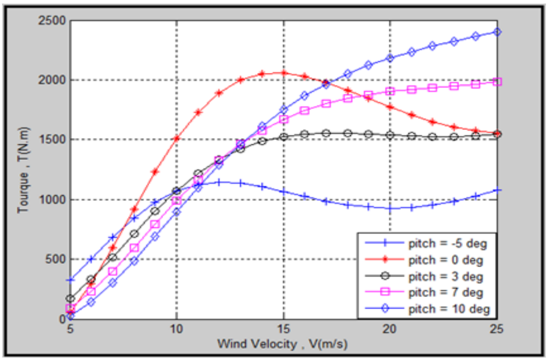



**Figure (10):** the relationship between Torque and wind velocity for different pitch angles.

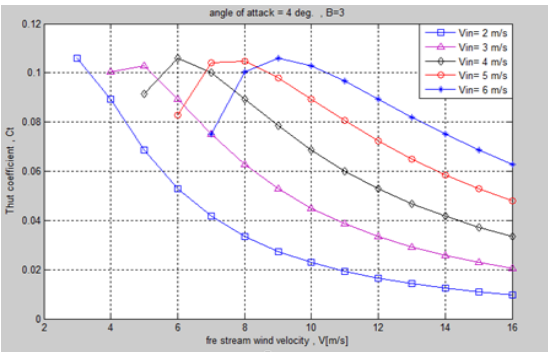



**Figure (11):** The relationship between power Coefficient and free stream wind velocity for different cut in wind velocities.

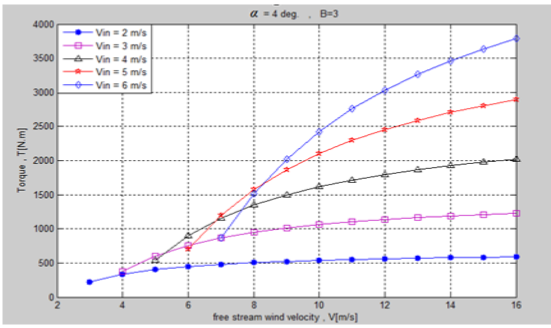



**Figure (12):** The relationship between torque and free stream wind velocity for different cut in wind velocities.

## References

**[1] Tony Burton**, " Wind energy Handbook : Design of wind turbine " , Johns Wiley and sons, 2001 .

**[2] S.Rajakumar., D.Ravindran** ;" Iterative approach for optimising coefficient of power, coefficient of and drag of wind turbine rotor", Elsevier Ltd, 2011.

**[3] Isaac Y., Joseph C.**,"A study of weibull parameters using long-term wind observations ", Renewable energy journal, Vol. 20,Issue 2.pp.145-153,Dec 1999.

**[4] someers**, "Summary of the Delft University Wind Turbine Dedicated Airfoils ", J.Sol.Energy Eng., Vol. 125, Issue 4, pp,488-497, November 2003 .

**[5] Doucette, Jessica S., Kenneth D. Visser**, " Solidity and blade number effects on fixed pitch, 50W horizontal axis wind turbine", Wind engineering Journal, Vol. 27, issue 4, pp.299-316,Aug 2009 .

**[6] sinopoli; Martin O., John E. Quaicoe**. "Wind Turbine Rotor modeling using response surface methodology", Energy Jornal, IEEE, vol. no. 3,Issue 2, pp.409-428, June 2009.

[7] HAWT Rotor Design and Performance Analysis, Proceeding of the ASME 2009 3rd International Conference of Energy Sustainability ES2009 July 19-23, 2009, San Francisco, California, USA.

**[8] Siraj Ahmed;** "wind Energy theory and practice ", second edition ; PHI leaving Ltd.; 2013.

**[9] Slootweg, J.G. ; Polinder**, H.; Kling,W.L.," Dynamic Modeling of wind turbine with doubly feed induction generator ", Power Engineering society Summer Meeting, 2001 . IEEE, vol.1, no., pp.644-649 vol.1, 2008.

**[10] A.H.J.A Martens & P.H.W.M Albers** "Wind Turbine study" October 2003.

**[11] Ragheb, M.**, Optimal rotor tip speed ratio, 2009, https://netfiles.uiuc. edu/mragheb/www/NPRE %20475%20 Wind%20Power%20Systems/Optimal%20Rotor%20Tip%20Speed%20Ratio.pdf

**[12] Fox, B., Flynn, D., Bryans, L., Jenkins, N., Milborrow, D., O'Malley, M., Watson, R. and Anaya-Lara, O.** Wind Power Integration: Connection and System Operational Aspects, IET Power and Energy Series 50, Institution of Engineering and Technology, Stevenage, ISBN 10: 0863414494. (2007)