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# Parametric Instability Regions of a MRE Cored Sandwich Beam Subjected to Periodic Axial Load

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***Abstract***

*In this work the dynamic stability of a magnetorheological elastomer (MRE) embedded viscoelastic cored sandwich beam to time varying axial load has been studied. The finite element method (FEM) is used to derive the governing equation of motion which is similar to that of Mathieu's equation. The instability regions of the sandwich beam for the principal parametric resonance case are investigated by using the harmonic balance method. The modal frequencies obtained from the analytical model have been compared with the previously published results. Effects of various parameters such as applied magnetic field, static load and the location of the MRE patch on the stability of the sandwich beam are investigated. The results suggest that the stability of the MRE embedded sandwich beams are influenced by the various system parameters.*

***Keywords:*** *Sandwich beam, Magnetorheological elastomer, conductive skin, dynamic instability, harmonic balance method.*

## 1. Introduction

Layered materials and sandwich structures are often used in structural systems to utilise the advantages of the different materials. The inclusion of different materials offers the possibility to combine specific material properties, and to improve mechanical properties while reducing the component weight. These have been received a great deal of attention because of their technologically interesting applications in many areas of engineering. Sandwich construction offers the structural designer many attractive features such as high specific stiffness, good buckling resistance, easy reparability, high corrosion resistance, good energy absorption capability, high fatigue life, buoyancy and lower maintenance cost, when compared to traditional complete metallic structures. Thus, the analysis of such structural systems has been investigated for long time due to these advantages. The most important applications are found in the transport industry such as in aerospaces, aircraft, automobiles, railroad, robot and marine industries where a high stiffness/weight and strength/weight ratio provides increased payload capacity, improved preference and lower energy consumption.

Sandwich structures are often subjected to various kinds of static and dynamic loading which give rise to severe vibration problems that not only affect the operation but also cause damage to components. The vibration can be controlled passively by structural modification or by actively by modifying the structural property with an external agency without changing the structure itself. However, it is difficult to use the passive control scheme when the frequency of vibration of the structures varies in a wide range. Therefore, it is essential to adopt smart materials to attenuate vibration by means of bonding or embedding them and creating composite or sandwich structures.

One of such material to improve the design of high stiffness and high strength sandwich structures is the use of magnetorheological elastomer (MRE) as core. Magnetorheological elastomers comprise of a class of smart materials whose rheological properties can be controlled rapidly and reversibly by the application of an external magnetic field. Sandwich beams with MRE cores possess field-controllable flexural rigidities due to the field-dependant shear modulus of the MRE core [1-3].

Sometimes these sandwich structures are subjected to axial periodic load and vibrate in transverse direction for some amplitude and frequency of external excitation. Such systems are generally called as parametrically excited systems as their governing equation of motion contains the periodic excitation as coefficient of the response of the system. The general description about parametrically excited system is given by Nayfeh and Mook, [4], Cartmell, [5]. In parametrically excited systems one may obtain the regions in the systems states space for which the systems become unstable. These regions are known as instability regions and there are many studies to find out these regions by using different techniques.

Magnetorheological elastomers (MRE) have great potential in developing stiffness variable devices which can find applications in many intelligent structures. These materials are increasingly being used as semi-active/active vibration devices in various applications [6-8]. The rheological properties of MREs such as damping and stiffness can be changed and controlled by varying magnetic field [9-11]. Chen et al [12] developed high modulus natural rubber based MREs by considering different percentage of iron particles and reported that the increase in weight fraction of iron particles increases the shear modulus of MRE.

When a beam is subjected to a time varying axial load the system behaves as that of a parametrically excited system [4]. In these systems one should study the parametric instability regions to obtain the critical system parameters to avoid excessive vibration of the system. Bolotin [13] studied the dynamic stability of the beams subjected to time varying axial compressive forces. The dynamic instability of sandwich structures induced by parametric excitation has been investigated by many researchers [14-16]. Zhou and Wang [17-19] studied the dynamic properties of sandwich beams with MRE embedded soft cores with nonconductive and conductive skins. Dwivedy et al [20] investigated the instability regions of a MRE embedded soft cored sandwich beam subjected to periodic axial load using higher order theory. The dynamic stability of a sandwich beam with MRE embedded viscoelastic core which is incompressible in transverse direction has been studied by Nayak et al [21]. In this present work an attempt has been made to develop a finite element based method to study complicated MRE embedded viscoelastic cored sandwich beam. The natural frequencies obtained using this method, have been compared with the published results. The instability regions are determined by solving the obtained Mathieu-Hill's equation using Harmonic balance method. The effects of static and dynamic loads, magnetic field strength and location and length of MRE segment on the instability regions are determined.

## 2. MRE Adaptive Sandwich Beam Model Using Finite Element Method Layout

Figure 1 shows the schematic diagram of a three layered MRE embedded viscoelastic cored sandwich beam of length L, top, bottom and core layers thickness $h_t$, $h_b$ and $h_c$, respectively. The core layer contains both the viscoelastic patches of lengths $L_1$ and $L_3$ and a MRE layer segment of length $L_2$. This system is subjected to a time varying axial force, $P(t) = P_s + P_d \cos \Omega t$ . Here $P_s$ and $P_d$ are the static and dynamic loads respectively. t is the time and $\Omega$ is the excitation frequency.The following assumptions are considered for the modeling of the sandwich beam using FEM. It is assumed that the deformation of top and bottom skins obeys Euler Bernoulli beam theory. The three layers have the same transverse displacement w. The core of the sandwich beam deforms due to shear only. The non-MRE parts of the core are not affected by magnetic field but only the MRE part of the core is affected by the magnetic field. The zero field Young's modulus and shear modulus are same for both MRE and non-MRE parts in the core. There is perfect bonding between the layers.

The strain in top and bottom skins can be expressed in terms of axial displacement and the transverse displacement as follows.

$$\frac{\partial u_j}{\partial x} = \frac{\partial u_{0j}}{\partial x} - z_j \frac{\partial^2 w}{\partial x^2} \tag{1}$$

where, subscript *j*= t and b for top and bottom faces, respectively, $u_{0j}$ is the axial displacement of the mid-plane of skin *j*, and $z_j$ is the distance of the mid-height of skin *j* from the neutral axis.

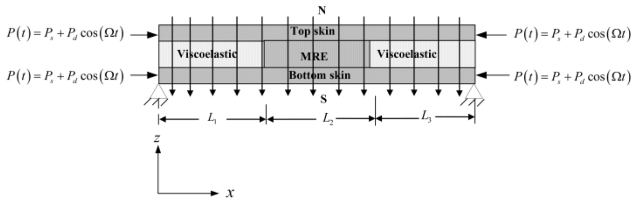



**Fig.1** MRE embedded viscoelastic cored sandwich beam subjected to time varying axial load.

As discussed in Mead and Markus [16] the expression for shear strain $\gamma_c$ can be given by

$$\gamma_c = \frac{H}{h_c}\frac{\partial w}{\partial x} + \frac{(u_{0t} - u_{0b})}{h_c} \tag{2}$$

where, $H = h_c + \frac{(h_t + h_b)}{2}$.

The total kinetic energy of the sandwich beam can be obtained by adding the kinetic energy due to the transverse displacement of all the layers, axial displacements of top and bottom skins and the rotation due to shear strain of the MRE embedded viscoelastic core.

$$T = \frac{1}{2}\int_0^L \left( m\left(\frac{\partial w}{\partial t}\right)^2 + m_t\left(\frac{\partial u_t}{\partial t}\right)^2 + m_b\left(\frac{\partial u_b}{\partial t}\right)^2 + \rho_c I_c \left( \frac{H}{h_c}\frac{\partial^2 w}{\partial x \partial t} + \frac{(\partial u_{0t}/\partial t) - (\partial u_{0b}/\partial t)}{h_c} \right)^2 \right) dx \tag{3}$$

where, $m = m_t + m_c + m_b$. $m_t$, $m_c$ and $m_b$ are the mass per unit length of the top, middle and bottom layers respectively, $\rho_c$ and $I_c$ are the density and the moment of inertia about centroid of the core, respectively.

The expression for potential energy of the system *U* can be obtained by adding the potential energy due to extension and bending of the skins, shear deformation of the core and work done due to the magnetoelastic loads in the skins which is given as follows.

$$U_{ebs} = \frac{1}{2}\int_0^L \left( E_t A_t \left(\frac{\partial u_t}{\partial x}\right)^2 + E_b A_b \left(\frac{\partial u_b}{\partial x}\right)^2 \right) dx + \frac{1}{2}\int_0^L \left(E_t I_t + E_b I_b\right)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$

$$+\frac{1}{2}\int_0^L G_c^* A_c \left[ \frac{H}{h_c}\frac{\partial w}{\partial x} + \frac{u_{0t} - u_{0b}}{h_c} \right] dx \qquad (4)$$

Here, the complex shear modulus $G_c^* = G_c(1 + i\eta_c)$, where $G_c$ is the storage shear modulus, $A_c$ is the cross sectional area of core, $i = \sqrt{-1}$ and $\eta_c$ is the core loss factor.

The non-conservative work done due to periodic axial load is:

$$W = \frac{1}{2}\int_0^L P(t)\left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (5)$$

A standard beam element with two end nodes (Fig.2) with four degrees of freedom (DOF) at each node is considered for modeling of the sandwich beam using FEM. The DOF include the transverse displacement w, axial displacement of top skin $u_t$, axial displacement of bottom skin $u_b$ and the rotational displacement $\theta$ of the beam. The elemental displacement vector is:

$$\left\{q^e\right\} = \left\{w^i, \theta^i, u_t^i, u_b^i, w^j, \theta^j, u_t^j, u_b^j\right\}^{\mathrm{T}} \qquad (6)$$

The elemental displacements can be determined in terms of displacements of two nodes as:

$$\{w u_t\, u_b\}^T = \{[N_w][N_{ut}][N_{ub}]\}^T \{q^e\} \qquad (7)$$

The shape functions are [34],

$$\left.\begin{aligned} [N_w] &= \left\{1 - \frac{3x^2}{l_e^2} + \frac{2x^3}{l_e^3} \;\; x - \frac{3x^2}{l_e} + \frac{x^3}{l_e^2} \;\; 0 \;\; 0 \;\; \frac{3x^2}{l_e^2} + \frac{2x^3}{l_e^3} \;\; -\frac{x^2}{l_e} + \frac{x^3}{l_e^2} \;\; 0 \;\; 0\right\} \\ [N_{ut}] &= \left\{0 \;\; 0 \;\; 1 - \frac{x}{l_e} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{x^2}{l_e} \;\; 0\right\} \\ [N_{ub}] &= \left\{0 \;\; 0 \;\; 0 \;\; 1 - \frac{x}{l_e} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{x^2}{l_e}\right\} \end{aligned}\right\} \qquad (8)$$

Upon substituting the expressions for kinetic energy T and potential energy U into Hamilton's principle, described as

$$\delta\int_{t_1}^{t_2} (L + W)dt = 0 \qquad (9)$$

where, $L = T - U_{ebs}$

The governing equations of motion for the un-damped partially or fully treated MRE sandwich beam element in the finite element form can be obtained as

$$[M]\{\ddot{q}\} + [K]\{q\} - (P_s + P_d\cos\Omega t)\left[K_f\right]\{q\} = 0 \qquad (10)$$

For the MRE embedded sandwich beam, the matrices $[M], [K]$ and $\left[K_f\right]$ are formulated by imposing compatibility conditions at the interfaces of the viscoelastic material and MRE patches within the core of the sandwich beam.

Considering the damping effect of MRE on the sandwich beam the equation of motion can be rewritten as.

$$[M]\{\ddot{q}\} + [K]\{q\} + [C]\{q\} - (P_s + P_d\cos\Omega t)\left[K_f\right]\{q\} = 0 \qquad (11)$$

where, $[C] = i\eta_c[K]$, here $\eta_c$ is the loss factor of the MRE.

## 3. Dynamic Stability Analysis

For the analysis of stability of sandwich beams the method developed by Bolotin [13] is applied to obtain the relation for the dynamic instability of the system. Considering the damping effect of MRE on the sandwich beam the equation of motion can be rewritten as.

The derived equation is a Mathieu-Hill equation with a periodic coefficient. The periodic motion of the system is usually the boundary case of vibrations with unboundedly increasing amplitudes. Therefore it is important to study the dynamic instability of the system and determination of the boundaries of the dynamic instability regions. The equation of boundary frequencie have been derived and given below.

$$\begin{vmatrix} [K] - (\alpha P_{cr} - \beta P_{cr})[K_f] - \frac{\Omega^2}{4}[M] & -\frac{\Omega}{2}[C] \\ \frac{\Omega}{2}[C] & [K] - (\alpha P_{cr} + \beta P_{cr})[K_f] - \frac{\Omega^2}{4}[M] \end{vmatrix} = 0 \qquad (12)$$

This above equation is used to find the boundaries of principal instability regions of the system.

## 4. Results and Discussions

A MATLAB code has been developed to obtain the natural frequency, loss factor and parametric instability regions of a simply supported MRE embedded sandwich beams for different configurations. The beam is divided into 32 elements for analysis. The developed code is validated by comparing the natural frequencies with the published results available in literature.

**Table 1** Comparison of modal frequencies (Hz) for cantilever beam

| | Modal frequencies (Hz) | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Present Analysis | 33.754 | 199.126 | 513.174 | 909.954 | 1355.30 |
| Howson and Zare | 33.754 | 198.992 | 512.307 | 907.299 | 1349.65 |

**Table 2** Comparison of modal frequencies (Hz) for cantilever beam

| | Modal frequencies (Hz) | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Present Analysis | 291.15 | 1692.1 | 4675.2 | 9125.5 |
| Banerjee | 291.50 | 1684.48 | 4623.98 | 8945.18 |

The natural frequencies obtained from the present analysis are compared with those obtained by Howson and Zare [22] and Banerjee et al [23] for cantilever beam as given in Table 1 and Table 2 respectively. The frequencies are found to be in good agreement with the published results.

### *4.1 Natural frequencies and loss factors*

In the present numerical analysis a symmetric sandwich beam with MRE embedded core has been considered for simply supported end conditions. The geometric and material properties of the sandwich beam are as follows. The span of the beam, $L$ = 415 mm; width, 27 mm; the top and bottom skins thickness, $h_t$=$h_b$=0.9 mm, the core thickness, $h_c$=2.9 mm. The top and bottom aluminum skins have Young's modulus 72 GPa and density 2700 kg/m3. Following expressions for the shear storage modulus and loss factor of natural rubber based MRE (containing 80% of iron particles) have been used which are obtained by curve fitting the experimentally obtained data of Chen et al [12] up to the saturated magnetic field strength of 0.6 T.

$$G_c = \left(-6.9395B_0^6 - 9.1077B_0^5 + 71.797B_0^4 - 93.422B_0^3 + 38.778B_0^2 + 2.43B_0 + 2.7006\right)\text{MPa}$$

$$\eta_c = 5.3485B_0^6 - 17.787B_0^5 + 22.148B_0^4 - 12.185B_0^3 + 2.3522B_0^2 + 0.1526B_0 + 0.228$$

In this work along with the full length MRE core sandwich beam five other different types of MRE embedded sandwich beam configurations shown in figure 2 have been considered for numerical analysis. The five different sandwich beam configurations CI, LI, LII, LIII and LIV have the same MRE patch length but with different MRE patch locations.

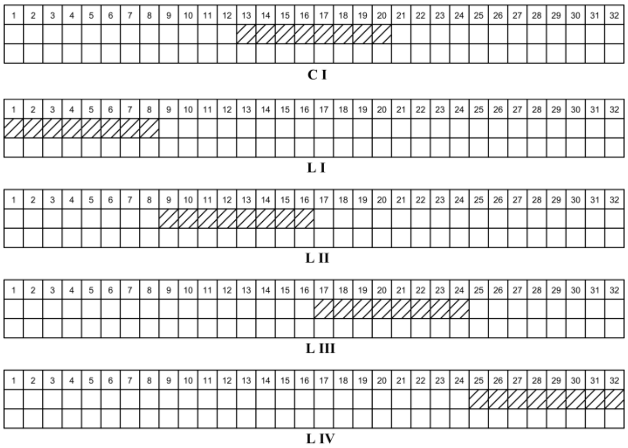



**Fig. 2** Different locations of MRE patches in the core

Fig. 3 shows the effect of variation of magnetic field strength on the fundamental frequency and loss factor for the different configurations of the sandwich beam shown in Fig. 3. Also the results have been compared with those of the configuration CI and the fully treated MRE core of sandwich beam. One may observe that due to symmetry, the fundamental frequencies and loss factors for location LI and LIV and also for LII and LIII are same as shown in Fig. 3 (a) and (b) respectively. Also it can be observed that at higher magnetic field the fundamental frequencies and loss factors of LI and LIV in which the MRE patches are located at the support ends are more than those of LII and LIII.

### *4.2 Parametric instability regions*

In this subsection, the stability of a MRE embedded viscoelastic cored sandwich beam subjected to periodic axial load has been investigated considering various system parameters and different configurations based on the length and location of the MRE patch in the core for the simply supported end condition.

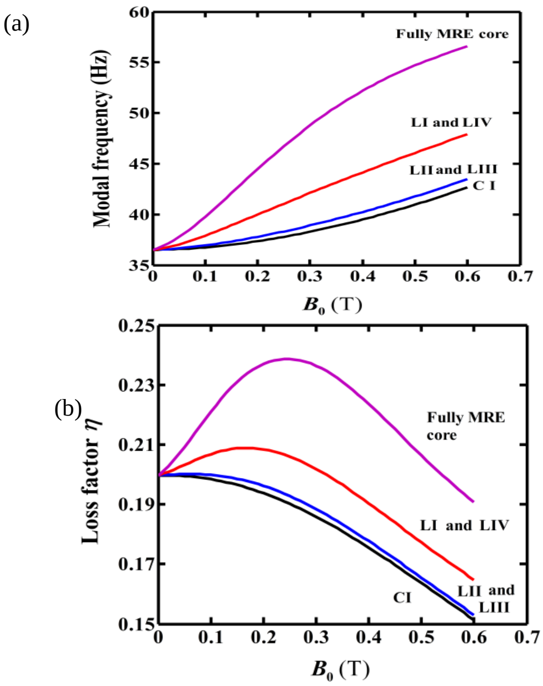
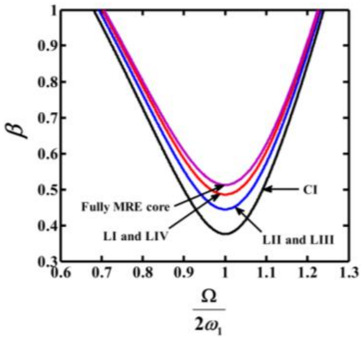
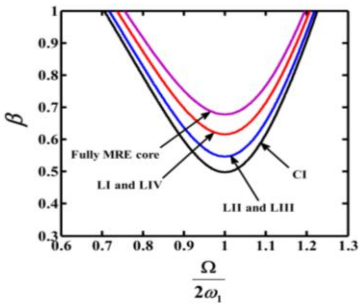



**Fig. 3** Variation of (a) fundamental frequency and (b) loss factor with magnetic field for different configurations CI, LI, LII, LIII, LIV and fully treated MRE core sandwich beam (P(t)=0)).

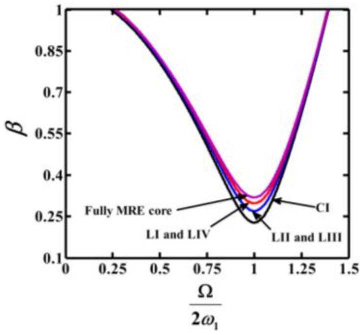
Figure 4 shows the influence of location of the MRE patch on the stability of the sandwich beam which has been obtained for four different configurations, LI, LII, LIII and LIV (Fig.4). Also these results are compared with those obtained for the configuration CI and fully MRE cored sandwich beam. Comparing the Fig. 4 (a) and (b), the instability regions decreases with increase in magnetic field for all the locations of the sandwich beam. From Fig.4 (c) and (d) one may observe that with increase in static load factor α, while the width of the instability regions increases the value of $\beta_{cr}$ decreases making the system more unstable. One may observe that due to symmetry the instability regions of simply supported end condition are same for location LI and LIV and also for LII and LIII. For the same system parameters the instability regions of locations LI and LIV are less than that of the locations LII and LIII. This is because that stiffness and damping capacity of the locations LI and LIV increase due to the location of MRE patches at the boundary edges of the simply supported sandwich beam as presented in Fig. 2.



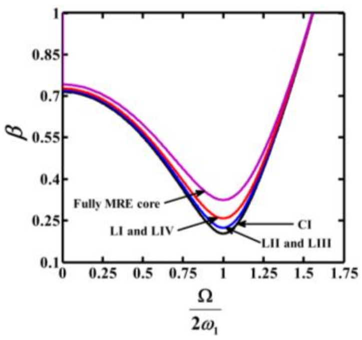

(a)




(b)




(c)




(d)

**Fig. 4** Dynamic principal instability regions of a sandwich beam with different locations of MRE patch (a) $\alpha = 0, B_0 = 0.2\mathrm{T}$, (b) $\alpha = 0, B_0 = 0.6\mathrm{T}$, (c) $\alpha = 0.4, B_0 = 0.2\mathrm{T}$ and (d) $\alpha = 0.6, B_0 = 0.6\mathrm{T}$.

## 5. Conclusions

In this paper, the instability regions of a sandwich beam have been investigated for the principal parametric resonance condition. The finite element method has been used for mathematical modeling of the MRE embedded sandwich beam. The comparison of the results obtained herein with those in the previous literatures indicated that the natural frequencies and loss factors can be predicted with considerable accuracy using the method presented.

Analysis has been made for five different configurations of the sandwich beam by varying the location. It has been observed that the instability regions of the sandwich beam can be changed by varying position of the MRE patches in a viscoelastic core. So the system stability can be achieved passively by changing the location of the MRE patches in the core and actively achieved by applying magnetic field of suitable amplitude. This formulation can be used for developing stiffness variable devices for vibration reduction and for the sandwich structures with complicated geometry.

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