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Extreme Value Analysis to Determine the Probable Maximum Wind Speed at Mersa Matruh Coastal Area

M. A. Sadek and O.S. Ahmed

Nuclear and Radiological Regulatory Authority, Cairo Egypt

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Abstract

A daily midnight value of wind speed at Mersa Matruh coastal area has been collected for 30 years , from 1980 to 2012. The maximum yearly wind speed values have been determined and used for the extreme analysis, using the extreme value distribution type 1 (EV1) which is commonly known as Gumbel's distribution . The graphical approach and parametric analytical one have been used for fitting the distribution of empirical data to the mathematical equation of the theoretical function, to make predictions beyond the period of observation. A quite good match exists between the results of graphical and parametric approaches. The equations of the best fit trend lines may express an identity to the study area, in a way to be used confidently for prediction purposes. The extreme wind speed that can affect Mersa Matruh coastal area , while having a probability of non exceedence in a given year, equals 2%, 1%, 0.05%, 0.002% or 0.001, has been determined. For safety considerations the structures which are exposed to wind are designed to withstand the effect of the extreme wind speed of appropriate exceedence probability. The exceedence probability for which the Probable Maximum Wind Speed (PMWS) values have been determined in this work, are chosen to cover a wide range of structure criticality design basis.

***Keywords**: Gumbel's Distribution Function, Maximum Wind Speed, Method of Moment, Graphical, Analytical Parametric Approach**,** Mersa Matruh*

Introduction

High wind speed poses a threat to the integrity of structures particularly those at exposed sites . For safety considerations the structures are designed to withstand the effects of wind speed of appropriate recurrence , exceedence probability and accepted risk. Accurate assessment of the magnitude and frequency of extreme wind speed is of fundamental importance for many safety, engineering and financial applications , the extreme value analysis are used for that . The criticality of the structure to be protected from the effects of extreme wind speed determines the return period for which the extreme wind should be determined to protect the structure against [1,2]. . The present study is devoted to predict the extreme wind speed (probable maximum wind speed) which is expected to affect the coastal area of Mersa Matruh , under different exceedence probabilities or return periods . The extreme value analyses approach has been used for that.

Methodology

The extreme value distribution, type 1 (EV1) was introduced by Gumbel and commonly known as Gumbel's distribution function [3]. It is one of the most common functions used for frequency analysis and for modeling the extreme values (maximum or minimum) . It is widely used around the world for analysis of extreme hydrologic and meteorologic values and for predicting the maximum highly conservative range for the design safety concern. The Gumbel distribution is the most appropriate probabilistic model to describe extreme wind speed behavior. Its use is based on determining a mathematical formula for an empirically obtained frequency distribution (e.g. from the observed wind speed data series). This is done either graphically or analytically . The graphical method [4,5 and 6] depends on plotting the ascendingly or descendingly ranked maximum or minimum wind speed values against their cumulative probabilities, excceedance probabilities (expressed as Gumbel reduce variate), or return periods . The analytical method on the other hand relies on fitting a theoretical frequency distribution function expressed by mathematical equation to the data series of the maximum or minimum wind speed values. Several methods are used for fitting the observed data to the theoretical distribution the method of moment (MOM)[7] which is used in the current work is one of them. It depends on matching the sample moments (mean,

Table (1) The Maximum wind speed, Average and Standard deviation (Mersa Matruh area, 1980 - 2012)

Years	Maximum wind speed (Knots)	Years	Maximum wind speed (Knots)	Years	Maximum wind speed (Knots)
1980	35	1992	25.5	2004	25.7
1981	26	1993	27.8	2005	25
1982	21	1994	24.4	2006	25.2
1983	30	1995	26.4	2007	29.1
1985	36.4	1996	26.2	2008	26.2
1986	31.5	1998	26	2009	29.5
1987	23.8	2000	29.2	2010	24.7
1988	31.5	2001	24	2011	25.9
1990	26.1	2002	26.3	2012	27.6
1991	32.9	2003	32.9	**Means and Standard deviation**	**27.59 and 3.47**

Table (2) Frequency analysis, based on interval of maximum daily wind speed

(wind speed ,Knots) (Depth interval)	Mi (Number of observations)	mi/n (Frequency)	M/n (Exceedance frequency)	1-(M/n) (Cumulative frequency)	1/(M/n) (Return period)
21-24	2	0.066667	1	0	1
24-27	16	0.533333	0.93333333	0.066667	1.071429
27-30	5	0.166667	0.4	0.6	2.5
30-33	5	0.166667	0.23333333	0.766667	4.285714
33-36	1	0.033333	0.06666667	0.933333	15
36-39	1	0.033333	0.03333333	0.966667	30

standard deviation) to the corresponding moments of the theoretical distribution. Moments estimators related to the shape, frequency or scale theoretical distribution are used for fitting The fitted distribution is not only used to interpolate, but also to extrapolate, i.e. to find return periods of extreme values that were not apparent during the relatively short period of observation. The long data series increases the reliability of extrapolation and the conformance between the distribution of data and the theoretical one chosen.

Results and Discussion

A daily midnight value of wind speed at Mersa Matruh coastal area has been collected from National Ocean Atmospheric Administration (NOAA) [8] for 30 years from 1980 to 2012 . The maximum yearly wind speed values have been determined as indicated in Table (1), These are used for the analysis of extremes, using graphical approach and parametric analytical one .

Graphical *Approach*

The graphical regression approach has been used based on interval and ranking methodologies. In the interval methodology of the graphical approach, the maximum yearly wind speeds are ranked and intervaled and the cumulative frequency of occurrence of the average value of each interval are determined as reflecting the probability of its exceedence, Table (2). The relation between average value of each interval and its exceedence probability has been constructed as indicated in Fig. (1). The return period of each wind speed value has been determined (as the reciprocal of the exceedence probability) and plotted against the corresponding wind speed, Fig. (2). The best fit equations of the two relationships are calculated and indicated in the two figures . These equations have been used for calculating the maximum wind speed that can be expected to occur in the area of study, at the variant return periods.

In the ranking methodology of the graphical approach, the maximum yearly wind speed data for the study area is arranged in descending orders. The Gringorten formula, Q = (i-c1)/(N+c2) , the Bloom's formula Q = (m-(3/8))/(N+(1/4)) and the Weibull formula Q = (i)/(N+1) have been used to calculate the probability of exceedence of wind speed, Q, (where 'i' is the rank order and 'N' is the total number of values (30, for the thirty years), c1 = 0.44 and c2 = 0.12) . The reduced variate (y) has also been calculated for each of the Q values , it equals y = -In [-In (1-Q)]). Table (3) shows the results of calculation of ranking methodology of graphical approach.

The maximum wind speed is then plotted against the exceedence probability, the return period and the reduced variate of Q Figs. (3, 4 and 5) . The best fit equations of the three relationships are calculated and indicated in the figures . These equations can be used for calculating the maximum wind speeds that can be expected to occur in the area of study, at the variant return periods based on Ranking Methpodology as has been done for Interval Methodology, Table (5).

Table (3) Exceedence Probability and Gumbel Reduced Variate, based on Ranking Methodology of Graphical Approach

rank	wind speed (knots)	Bloom's formula	Gringorten formula	Weibull formula	Reduced variate of Bloom's formula	Reduced variate of Gringorten formula	Reduced variate of Weibull formula
1	36.4	0.02066116	0.018592	0.032258	3.869079183	3.97563892	3.417637092
2	35	0.05371901	0.051793	0.064516	2.896507537	2.934030294	2.707679652
3	32.9	0.08677686	0.084993	0.096774	2.39937109	2.421098882	2.284915186
4	32.9	0.11983471	0.118194	0.129032	2.058497915	2.073196456	1.979412778
5	31.5	0.15289256	0.151394	0.16129	1.796202844	1.806909006	1.73789269
6	31.5	0.18595041	0.184595	0.193548	1.581171219	1.589290738	1.53659934
7	30	0.21900826	0.217795	0.225806	1.397595131	1.403892121	1.362838126
8	29.5	0.25206612	0.250996	0.258065	1.236355872	1.241290611	1.209008835
9	29.2	0.28512397	0.284197	0.290323	1.091697854	1.095567957	1.07018592
10	29.1	0.31818182	0.317397	0.322581	0.959740519	0.962748467	0.942981875
11	27.8	0.35123967	0.350598	0.354839	0.837729304	0.840018033	0.824954504
12	27.6	0.38429752	0.383798	0.387097	0.723624078	0.725297145	0.714272302
13	26.4	0.41735537	0.416999	0.419355	0.615856868	0.61699051	0.609513182
14	26.3	0.45041322	0.450199	0.451613	0.513180732	0.51383138	0.509536687
15	26.2	0.48347107	0.4834	0.483871	0.414570453	0.414779527	0.413398773
16	26.2	0.51652893	0.5166	0.516129	0.319153763	0.318950734	0.32029204
17	26.1	0.54958678	0.549801	0.548387	0.226160794	0.22556508	0.229501376
18	26	0.58264463	0.583001	0.580645	0.134884068	0.133906043	0.140368602
19	26	0.61570248	0.616202	0.612903	0.044643628	0.043284879	0.0522616
20	26	0.64876033	0.649402	0.645161	-0.0452472	-0.046994386	-0.035455877
21	25.9	0.68181818	0.682603	0.677419	-0.13552018	-0.137674265	-0.12345767
22	25.7	0.71487603	0.715803	0.709677	-0.227001079	-0.229594072	-0.212497181
23	25.5	0.74793388	0.749004	0.741935	-0.320679515	-0.323761954	-0.303466094
24	25.2	0.78099174	0.782205	0.774194	-0.417819025	-0.421468895	-0.39748472
25	25	0.81404959	0.815405	0.806452	-0.520147185	-0.524486662	-0.496053695
26	24.7	0.84710744	0.848606	0.83871	-0.630217931	-0.635447503	-0.601332993
27	24.4	0.88016529	0.881806	0.870968	-0.752190268	-0.758667497	-0.716713717
28	24	0.91322314	0.915007	0.903226	-0.893805948	-0.902265697	-0.848172442
29	23.8	0.94628099	0.948207	0.935484	-1.072948564	-1.085359383	-1.008264451
30	21	0.97933884	0.981408	0.967742	-1.355706231	-1.382539294	-1.233722036

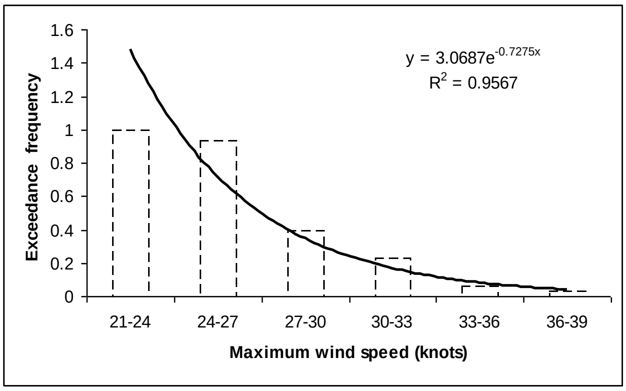


Figure (1) Relationship of Exceedence Probability and Maximum Wind Speed, (Interval Method of Graphical Approach)

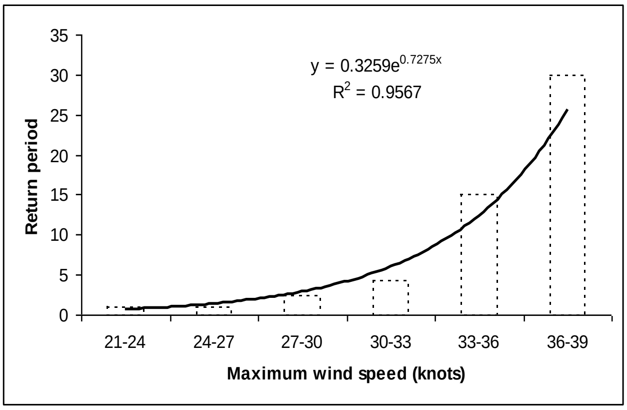


Figure (2) Relationship of Return Period and Maximum Wind Speed, (Interval Method of Graphical Approach)

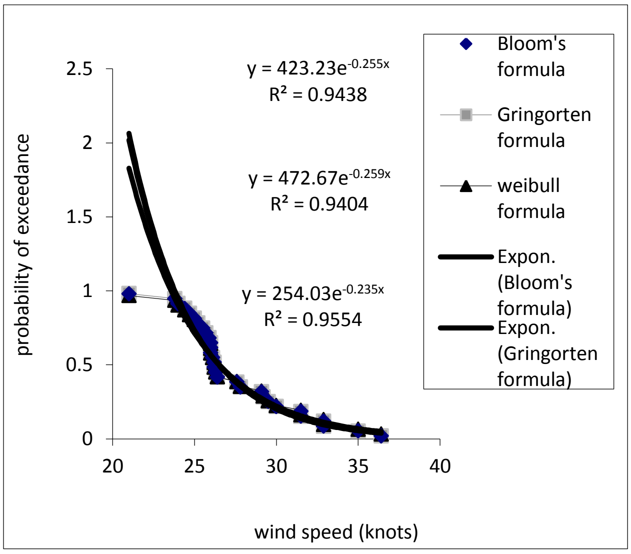


Figure (3) Relationship of Exceedence Probability and Maximum Wind Speed, (Ranking Methodology of Graphical Approach)

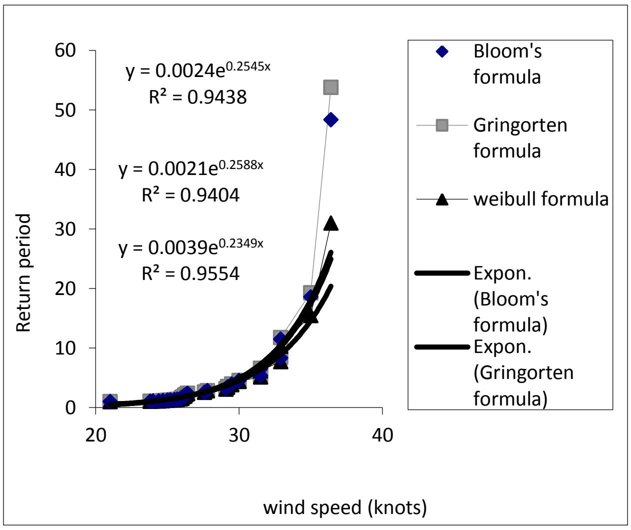
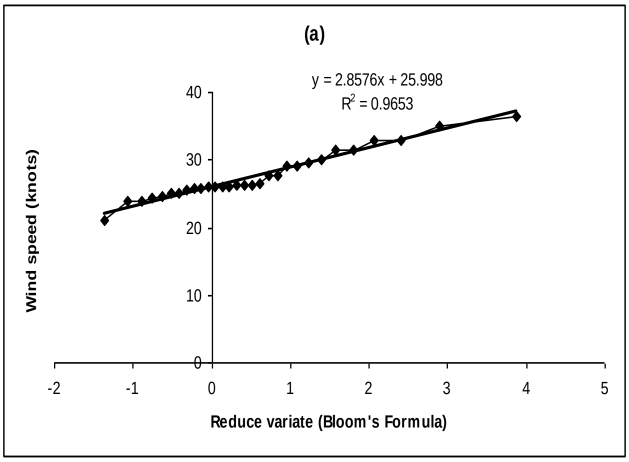


Figure (4) Relationship of Return period and maximum wind speed, (Ranking Methodology of Graphical Approach)



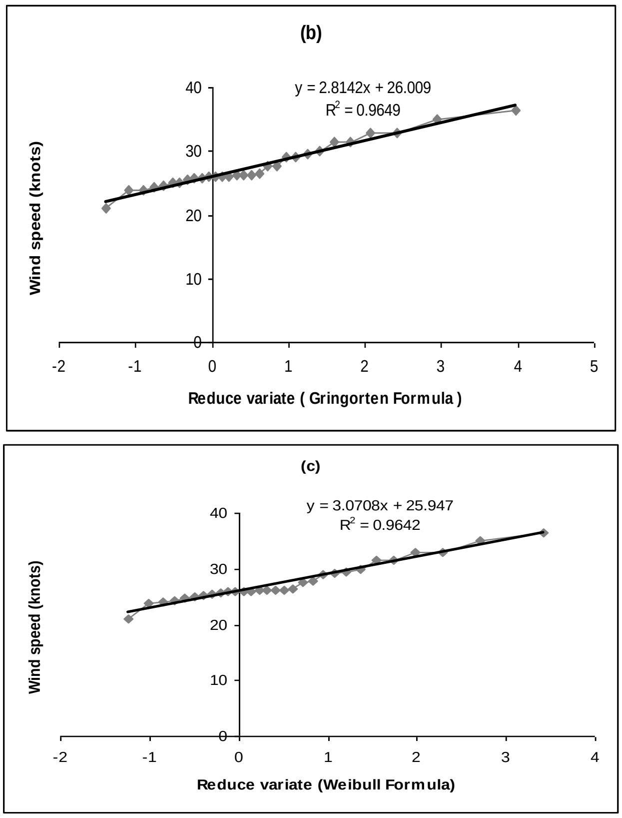


Figure (5) Analysis of annual maximum wind speed, using different formula (a- Bloom's formula & b- Gringorten formula and c -Weibull formula)

The Analytical Parametric Approach

The mean and standard deviation of the maximum yearly wind speed of the collected data have been calculated, Table (1). These values have been used to fit the mathematical expression of the Gumbel distribution function to aid for extrapolation and prediction beyond the period of observation of data collected. Moment estimators (u and α) which are specific to the shape and frequency of the Gumbel distribution function are used for fitting purposes. The Gumbel Distribution Function is expressed mathematically as follow:

$$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$$

$$\alpha = \frac{\sqrt{6}s_x}{\pi} \qquad u = \bar{x} - 0.5772\alpha$$

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right]$$

Where (x) is the extreme wind speed value for which exceedence frequency (probability) function is f (x) and F (X). The parameter estimation technique is based on matching the sample moments (mean, standard deviation, coefficient of variation) to the corresponding moments of the theoretical distribution. Matching is performed using moment estimators of the Gumbel distribution (i.e u and α).

Table (4) Results of Extreme Value Analysis, Analytical Parametric Approach

rank, r	wind speed knots(descending) ,xr	F(x > xr) ,r/(n+ 1)	F(x ≤ xr), (1- r/(n+ 1))	Tr(years), (n+ l)/r	Kt	x(kt)	yt	X(yt)
1	36.4	0.032258065	0.967741935	31	2.215803	46.47423	3.417637	35.30143
2	35	0.064516129	0.935483871	15.5	1.661971	41.75501	2.70768	33.37482
3	32.9	0.096774194	0.903225806	10.33333333	1.332175	38.94482	2.284915	32.22756
4	32.9	0.129032258	0.870967742	7.75	1.093855	36.91409	1.979413	31.39852
5	31.5	0.161290323	0.838709677	6.2	0.905447	35.30867	1.737893	30.7431
6	31.5	0.193548387	0.806451613	5.166666667	0.74842	33.97063	1.536599	30.19686
7	30	0.225806452	0.774193548	4.428571429	0.61287	32.81561	1.362838	29.72532
8	29.5	0.258064516	0.741935484	3.875	0.42869	31.79308	1.209009	29.30787
9	29.2	0.290322581	0.709677419	3.444444444	0.384575	30.8703	1.070186	28.93115
10	29.1	0.322580645	0.677419355	3.1	0.285344	30.02475	0.942982	28.58596
11	27.8	0.35483871	0.64516129	2.818181818	0.193271	29.2402	0.824955	28.26566
12	27.6	0.387096774	0.612903226	2.583333333	0.106929	28.50448	0.714272	27.96531
13	26.4	0.419354839	0.580645161	2.384615385	0.025207	27.80812	0.609513	27.68102
14	26.3	0.451612903	0.548387097	2.214285714	-0.05278	27.14356	0.509537	27.40972
15	26.2	0.483870968	0.516129032	2.066666667	-0.12778	26.50452	0.413399	27.14883
16	26.2	0.516129032	0.483870968	1.9375	-0.20041	25.88562	0.320292	26.89616
17	26.1	0.548387097	0.451612903	1.823529412	-0.27124	25.28212	0.229501	26.64978
18	26	0.580645161	0.419354839	1.722222222	-0.34077	24.68964	0.140369	26.4079
19	26	0.612903226	0.387096774	1.631578947	-0.4095	24.10397	0.052262	26.16881
20	26	0.64516129	0.35483871	1.55	-0.47793	23.5209	-0.03546	25.93077
21	25.9	0.677419355	0.322580645	1.476190476	-0.54658	22.93594	-0.12346	25.69196
22	25.7	0.709677419	0.290322581	1.409090909	-0.61604	22.34407	-0.2125	25.45033
23	25.5	0.741935484	0.258064516	1.347826087	-0.687	21.73939	-0.30347	25.20347
24	25.2	0.774193548	0.225806452	1.291666667	-0.76034	21.11443	-0.39748	24.94833
25	25	0.806451613	0.193548387	1.24	-0.83724	20.45922	-0.49605	24.68085
26	24.7	0.838709677	0.161290323	1.192307692	-0.91936	19.75941	-0.60133	24.39515
27	24.4	0.870967742	0.129032258	1.148148148	-1.00937	18.99246	-0.71671	24.08204
28	24	0.903225806	0.096774194	1.107142857	-1.11192	18.11863	-0.84817	23.7253
29	23.8	0.935483871	0.064516129	1.068965517	-1.23681	17.05447	-1.00826	23.29086
30	21	0.967741935	0.032258065	1.033333333	-1.41269	15.55581	-1.23372	22.67903

The moment estimators can be combined using the equation

$$y = \frac{x-u}{\alpha}$$

as reduced variate in the Gumbel distribution function, the equation accordingly takes this form:

$$F(x) = \exp[-\exp(-y)]$$

$$y = -\ln[-\ln(F(x))] = -\ln[-\ln(1-p)] \quad where\ p = P(x \geq x_T)$$

$$y_T = -\ln\left[-\ln\left(1-\frac{1}{T}\right)\right]$$

$$x_T = u + \alpha y_T$$

Where: p is the probability of exceedence, xt is the extreme wind speed of a return period T

The calculated mean and standard deviation of the data of maximum yearly wind speed during the thirty years equal 27.59333 and 3.478691 respectively , Table (1) . Substituting these values in the equations of Gumbel moment estimators (u and α), values of 26.02699 and 2.7137 have been calculated for the two estimators respectively.

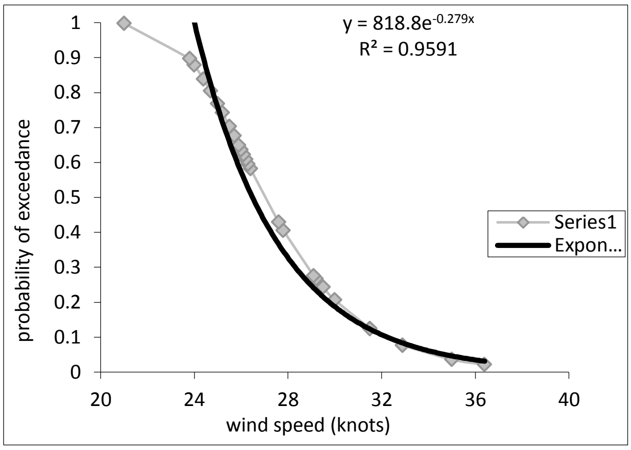


Figure (6) Relationship of exceedence probability and Maximum Wind Speed ,(Analytical Parametric Approach)

Table (5) Maximum wind speed at different return periods (Comparison between different methods of analysis)

Tr(years)	Maximum wind speed (knots)						
	Ranking			Gumbel distribution	Reduced Variate , Gumbel	Reduced Variate , Gringorten	Reduced Variate , Bloom's
	Weibull formula	Gringorten formula	Bloom's formula				
50	39.07391	42.90269	40.26736	38.0725	32.263	31.797	56.330
100	41.79747	45.8535	43.21817	40.55334	33.194	32.651	64.214
200	44.52104	48.80432	46.16899	43.03418	34.121	33.501	72.068
300	46.11422	50.53044	47.89511	44.48538	34.663	33.997	76.656
400	47.2446	51.75514	49.11981	45.51503	35.048	34.350	79.909
500	48.12139	52.70509	50.06976	46.31368	35.345	34.623	82.431
600	48.83778	53.48126	50.84593	46.96623	35.589	34.846	84.491
700	49.44348	54.1375	51.50217	47.51795	35.795	35.034	86.233
800	49.96817	54.70596	52.07063	47.99587	35.973	35.197	87.742
900	50.43097	55.20738	52.57205	48.41742	36.130	35.341	89.073
1000	50.84496	55.65591	53.02058	48.79452	36.271	35.470	90.263

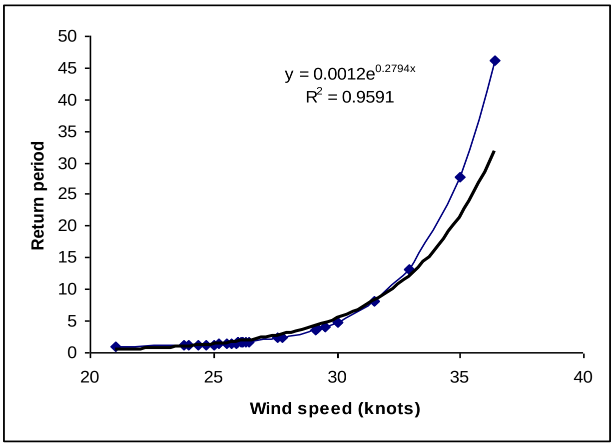


Figure (7) Relationship of Return Period and Maximum Wind Speed, (Analytical Parametric Approach

Fitting these values to the equation of the Gumbel probability distribution function, the exceedence probabilities (and accordingly the return periods) of the maximum wind speed values have been estimated based on analytical parametric approach as indicated in Table (4). Another way has been applied where the values of the moments estimators (u and α), have been combined within the reduced variate(y) values which are calculated and substituated in the Gumbel equation, to calculate the maximum wind speed at different recurrence., Table (4). The results of extreme values analysis, based on analytical parametric approach of Gumbel distribution are indicated in Figs (6 and 7)).

Table (5) has been established to compare between the results of the different methods used for analysis of extreme wind speed in the coastal area of Mersa Matruh. These show a comparison between the maximum wind speeds that can be expected at variant return periods (i.e. 50 ,100, 200, 300, 400, 500, 600, 700, 800, 900, 1000 years) , as calculated by ranking graphical methodology and analytical parametric method one.

Table (6) and Figure (8) have been constructed to illustrate the major trend of the analysis conducted. The maximum wind speed calculated using graphical method and analytical ones are highly comparable , the highest of which can be used for conservativety **.** The average values at 50 , 100, 500, 1000 , return periods ar**e** 38.57, 41.17, 47.21, 49.81 knots, respectively (about 19 m/s). The standard deviation of the values calculated (for the same return period) by graphical and analytical methods are low reflecting the high match and comparability between the methods. The Weibull equation is the highest comparable to the Gumbel one.

Table (6) Maximum wind speed at different return periods (comparison between Graphical Weibull method and Analytical Gumbel Method)

Return periods	50	100	500	1000
Weibull formula	39.07391	41.79747	48.12139	50.84496
Gumbel distribution	38.0725	40.55334	46.31368	48.79452
AVERAGE	38.57321	41.17541	47.21754	49.81974
Standard deviation.	0.500705	0.622065	0.903855	1.02522

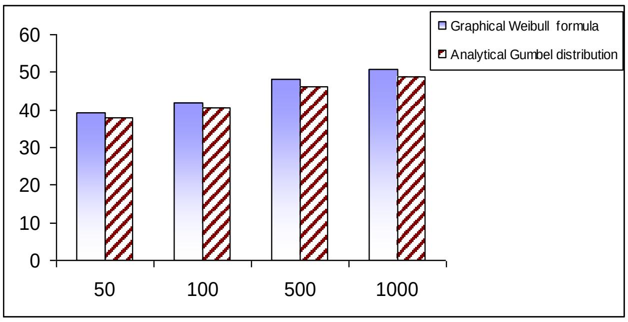


Figure (8) Comparison between maximum wind speed at different return periods (50, 100, 500, 1000 years), as calculated by graphical and analytical methods

Conclusion

The methodology of extreme value analysis has been employed for determining the probable maximum wind speed (PMWS) that can affect Mersa Matruh coastal area. The return periods and the corresponding probability of exceedence for which the PMSW values have been determined are chosen to cover a wide range of structure criticality design basis . A quite good match exists between the results of graphical and parametric approach. Their best fit regressions may express an identity to the study area, in the way to be used for prediction purposed

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