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Weakly non-linear magneto-convection in a viscoelastic fluid saturating a porous medium

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Abstract

In this paper the effect of magnetic field on linear and non-linear thermal instability in an anisotropic porous medium saturated with viscoelastic fluid is considered. Normal mode technique is used to investigate the linear stability analysis, while non-linear stability has been done using minimal representation of truncated Fourier series involving only two terms. Extended Darcy model, which includes the time derivative and magnetic field terms has been employed in the momentum equation. The criteria for both stationary and oscillatory convection are derived analytically. The effect of magnetic field is found to inhibit the onset of convection in both stationary and oscillatory modes. Thermal Nusselt number is defined in weakly non-linear stability analysis. Steady and transient behavior of the thermal Nusselt number is obtained by solving the finite amplitude equations using Runge-Kutta method. The nature of streamlines, Isotherms and Magnetic stream functions also has been investigated. The results have been presented graphically and discussed in detail.

Keywords: *Viscoelastic Fluid, Darcy-Rayleigh number, Darcy-Chandrasekhar number, Magnetic field, Porous medium, Anisotropy, Heat transfer, Nusselt number.*

Nomenclature

Latin Symbols

a	Wave number
a_c	Critical wave number
d	Depth of the porous layer
Da	Darcy number, $\left(K_z/d^2\right)$
g	Gravitational acceleration
H	Magnetic field $(H1, H2, H3)$
K	Inverse Permeability, $K_x^{-1}(\hat{i}\hat{i}+\hat{j}\hat{j})+K_z^{-1}(\hat{k}\hat{k})$
l, m	Horizontal wave numbers
p	Pressure
Pm	Magnetic Prandtl number, $\left(\Lambda/\kappa_{Tz}\right)$
$\Pr$	Prandtl number, $\left(\nu/\kappa_{Tz}\right)$
q	Velocity of the fluid (u, v, w)
Q	Darcy-Chandrasekhar number, $\left(\mu_m H_b^2 K_z/\rho_0 \nu \Lambda\right)$
Ra	Darcy-Rayleigh number $\left(\alpha_T g K_z (\Delta T) d/\nu \kappa_{Tz}\right)$
Ra_c	Critical Darcy-Rayleigh number
t	Time
T	Temperature
Va	Vadasz number, $\dfrac{\delta \Pr}{Da}$
$\overline{H}$	Rate of Heat transport per unit area
ΔT	Temperature difference between the walls
$\left(\rho c_p\right)_f$	Heat capacity of fluid
$\left(\rho c_p\right)_s$	Heat capacity of solid
$\left(\rho c_p\right)_m$	Relative heat capacity of the porous medium, $\delta\left(\rho c_p\right)_f+(1-\delta)\left(\rho c_p\right)_s$

Greek symbols

α_T	Thermal expansion coefficient
χ	Non-dimensional number, $(\delta Da/\Pr)$
δ	Porosity
η	Thermal anisotropy parameter $\left(\kappa_{Tx}/\kappa_{Tz}\right)$
γ	Ratio of heat capacities
κ_T	Thermal diffusivity $\kappa_{Tx}(\hat{i}\hat{i}+\hat{j}\hat{j})+\kappa_{Tz}(\hat{k}\hat{k})$
λ_1	Relaxation time

λ_2	Retardation time
Λ	Magnetic viscosity
μ	Dynamic viscosity
ν	Kinematic viscosity, (μ/ρ_0)
ω	Vorticity vector, $(\nabla\times \mathrm{q})$
ϕ	Magnetic Stream function
ψ	Stream function
ρ	Density
σ	Growth rate
ξ	Mechanical anisotropy parameter, (K_x/K_z)

Other symbols

b	Basic state
c	Critical
$*$	Non-dimensional value
$'$	Perturbed value
0	Reference state
$\hat{i}$	Unit normal vector in x-direction
j	Unit normal vector in y-direction
$\hat{k}$	Unit normal vector in z-direction
∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, horizontal Laplacian
∇^2	$\nabla_1^2+\frac{\partial^2}{\partial z^2}$
D	d/dz
i	$\sqrt{-1}$

1. Introduction

In the recent years, a great deal of interest has been focused on the understanding of the rheological effects occurring in the flow of non-Newtonian fluids through porous media. Many technological processes involve the parallel flow of fluids of different viscosity, elasticity and density through porous media. Such flows exist in packed bed reactors in the chemical industry, petroleum engineering, boiling in porous media and in many other processes. The flow through porous media is of considerable interest for petroleum engineers and in geophysical fluid dynamicists. Hence, the knowledge of the conditions for the onset of instability will enable us to predict the limiting operational conditions of the above processes. Excellent reviews of most of the findings on convection in porous medium are given by Nield and Bejan (2006), Ingham and Pop (2005) and Vafai (2006). However Horton and Rogers (1945) and Lapwood (1948) were the first to study the thermal instability in a porous medium. Many common materials such as paints, polymers, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. Flow and instability in viscoelastic fluids saturating a porous layer is of great interest in different areas of modern Sciences, engineering and Technology like material processing, petroleum, chemical and nuclear industries, Geophysics and Bio-mechanics engineering. Some oil sands contains waxy crudes at shallow depth of the reservoirs which are considered to be viscoelastic fluid. In these situations, a viscoelastic model of a fluid serves to be more realistic than the Newtonian model. Herbert (1963) and Green (1968) were the first to analyze the problem of oscillatory convection in an ordinary viscoelastic fluid of the Oldroyd type under the condition of infinitesimal disturbances. Later on Rudraiah et al. (1989, 1990) studied the onset of stationary and oscillatory convection in a viscoelastic fluid of porous medium. Kim et al. (2003) studied the thermal instability of viscoelastic fluids in porous media, conducted linear and non-linear stability analyses and obtained the stability criteria. Young-Yoon et al. (2004) studied the onset of oscillatory convection in a horizontal porous

layer saturated with viscoelastic fluid by using linear theory. Laroze et al. (2007) analyzed the effect of viscoelastic fluid on bifurcations of convective instability, and found that the nature of the convective solution depends largely on the viscoelastic parameters. Tan and Masuoka (2007) studied the stability of a Maxwell fluid in a porous medium using modifiedDarcy-Brinkman-Maxwell model, and found the criterion for onset of oscillatory convection.

Malashetty and Swamy (2007) studied the onset of convection in a viscoelastic liquid saturated anisotropic porous layer and obtained the stability criteria for both stationary and oscillatory convection. Sheu et al. (2008) investigated the chaotic convection of viscoelastic fluids in porous media and deduced that the flow behaviour may be stationary, periodic, or chaotic. However, the study of convective flow and instability in a porous medium under the influence of an imposed magnetic field has gained momentum during the last few decades due to its relevance and applications in engineering and technology. For example the above study is useful in commercial production of magnetic fluids. Other applications are in geophysics: to study the earth's core, where the molten fluid is viscoelastic and conducting, and becomes unstable due to differential diffusion; and to understand the performance of petroleum reservoir (Wallace et al. (1969)). Although the research field is quite interesting but only limited literature is available; Patil and Rudraiah (1973) have studied the problem of setting up of convection currents in a layer of viscous, electrically conducting fluid in the presence of a magnetic field, using linear and nonlinear theories, and investigated the combined effect of magnetic field, viscosity and permeability on the stability of flow through porous medium. Rudraiah and Vortmeyer (1978) have investigated the above problem for stability of finite-amplitude and overstable convection of a conducting fluid through fixed porous bed. Using linear and nonlinear analysis Rudraiah (1984) has studied the problem of magnetoconvection in a sparsely packed porous medium. Alchaar et al. (1995a, b) and Bian et al. (1996a, b) have also investigated the magnetoconvection in porous media for different physical models. Oldenburg et al. (2000) and Borglin et al. (2000) have carried out numerical and experimental investigations on the flow of ferrofluids in porous media. Sekar et al. (1993a, b) considered the problem of convective instability of a magnetized ferrofluid in a porous medium and studied the effect of rotation on it. Desaive et al. (2004) have studied linear stability problem of thermoconvection in a ferrofluid-saturating a rotating porous layer by considering Brinkman model and using modified Galerkin method, and discussed both stationary and overstable convections. Sunil et al. (2004, 2005) have investigated the effects of rotation and magnetic fields on thermosolutal convection in a ferromagnetic fluid saturating porous medium. Saravanan and Yamaguchi (2005) performed a linear analysis to study the influence of magnetic field on the onset of centrifugal convection in a magnetic fluid filled porous layer placed in zero-gravity environment and established the stability criterion. Recently Bhadauria (2008a) investigated the magnetoconvection in a porous medium under time dependent thermal boundary conditions. But in these entire studies porous medium is considered to be saturated by Newtonian fluid. To the best of authors' knowledge no literature is available in which magnetoconvection in a porous medium saturated by a viscoelastic has been investigated. Therefore the purpose of the present investigation is to study the effect of magnetic field on thermal instability in a porous medium saturated by a viscoelastic fluid. We obtain the result regarding the onset of convection using linear theory analysis and extract the informations for rate of heat transfer across the porous layer using a weakly nonlinear theory.

2. Governing Equations

We consider an electrically conducting viscoelastic fluid saturated horizontal anisotropic porous layer, confined between two parallel horizontal planes at $z=0$ and $z=d$, a distance d apart. The planes are infinitely extended horizontally in x and y directions. A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z- axis as vertical upward. A constant magnetic field is applied vertically upward across the porous layer. Adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperatures $T_0+\Delta T$, and T_0 respectively. Oberbeck Boussinesq approximation is applied to account the effect of density variations. The governing equations for magnetoconvection in a viscoelastic fluid saturating a porous medium are given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\delta}\frac{\partial q}{\partial t}-\mu_m H.\nabla H\right]+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu K.q =\left(1+\lambda_1\frac{\partial}{\partial t}\right)(-\nabla p+\rho g) \tag{1}$$

$$\gamma\frac{\partial T}{\partial t}+q.\nabla T=\nabla\left(\kappa_T.\nabla T\right) \tag{2}$$

$$\frac{\partial H}{\partial t}+(q.\nabla)\mathrm{H}-(\mathrm{H}.\nabla)\mathrm{q}=\Lambda\nabla^2 H \tag{3}$$

$$\nabla.q=0 \tag{4}$$

$$\nabla.\mathrm{H}=0 \tag{5}$$

$$\rho=\rho_0\left[1-\alpha_T(\mathrm{T}-\mathrm{T}_0)\right] \tag{6}$$

where $\gamma=\left(\rho c_p\right)_m\big/\left(\rho c_p\right)_f$ is ratio of heat capacities and $\kappa_T=\kappa_m\big/\left(\rho c_p\right)_m$ is effective thermal conductivity of porous media. . The thermal boundary conditions are

$$T=T_0+\Delta T \text{ at } z=0 \text{ and } T=T_0 \text{ at } z=d. \tag{7}$$

Eqs.(1)-(5) are satisfied by basic solution given by,

$$q_b=(0,0,0),\ \mathrm{p}=p_b(\mathrm{z}),\ \mathrm{H}=(0,0,H_b) \text{ and } \rho=\rho_b(\mathrm{z}) \tag{8}$$

We now slightly perturb the basic state and write

$$\mathrm{q}=\mathrm{q}_b+\mathrm{q}',\ T=T_b+\mathrm{T}',\ p=p_b+\mathrm{p}',\ \rho=\rho_b+\rho',\ \mathrm{H}=H_b+H' \tag{9}$$

Putting Eq.(9) in Eqs.(1)-(5) and using basic state Eq.(8), the perturbation equations are obtained in the form

$$\nabla.q'=0 \tag{10}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\delta}\frac{\partial q'}{\partial t}-\mu_m H_b.\nabla H'\right]+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu K.q'=\left(1+\lambda_1\frac{\partial}{\partial t}\right)(-\nabla p'+\alpha_T\rho_0 gT') \tag{11}$$

$$\gamma\frac{\partial T'}{\partial t}+(q'.\nabla)T+w'\frac{dT_b'}{dz}=\kappa_{Tx}\nabla_1^2 T'+\kappa_{Tz}\frac{\partial^2 T'}{\partial z^2} \tag{12}$$

$$\frac{\partial H'}{\partial t}+(q'.\nabla)\mathrm{H}'-(\mathrm{H}'.\nabla)\mathrm{q}'-\mathrm{H}_b\frac{\partial q'}{\partial z}=\Lambda\nabla^2 H' \tag{13}$$

For non-dimensionalization the following scaling has been used:

$$(\mathrm{x}',\ \mathrm{y}',\ \mathrm{z}')=(\mathrm{x}^*,\ \mathrm{y}^*,\ \mathrm{z}^*)\,\mathrm{d},\ \ \mathrm{t}=\frac{d^2}{\kappa_{Tz}}t^*,\ \ q'=\frac{\kappa_{Tz}}{d}q^*,$$

$$T'=(\Delta\,\mathrm{T})\,\mathrm{T}^*,\ p'=\frac{\mu\kappa_{Tz}}{K}p^*$$

$\lambda_1 = \frac{d^2}{\kappa_{Tz}}\lambda_1^*,\ \lambda_2 = \frac{d^2}{\kappa_{Tz}}\lambda_2^*$ and $H' = H_b H^*$

Then with the help of the above expressions the non-dimensionalized form of the above equations(dropping the asterisks for simplicity) is

$$\nabla.q = 0 \tag{14}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial q}{\partial t} - Q.P_m\frac{\partial H}{\partial z}\right] + \left(1+\lambda_2\frac{\partial}{\partial t}\right)q_a = -\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\nabla p - RaT\hat{k}\right) \tag{15}$$

$$\gamma\frac{\partial T}{\partial t} + (q.\nabla)T - w = \left(\eta\nabla_1^2 + \frac{\partial^2}{\partial z^2}\right)T \tag{16}$$

$$\frac{\partial H}{\partial t} + (q.\nabla)\mathrm{H} - (\mathrm{H}.\nabla)\mathrm{q} - \frac{\partial q}{\partial z} = P_m\nabla^2 H \tag{17}$$

where $Q = \frac{\mu_m H_b^2 K_z}{\rho_0 \nu \Lambda}$ is Darcy-Chandrasekhar number, $P_m = \frac{\Lambda}{\kappa_{Tz}}$ is the magnetic Prandtl number, $Ra = \alpha_T g K_z(\Delta \mathrm{T})\mathrm{d}/\nu\kappa_{Tz}$ is the Darcy-Rayleigh number, $q_a = \left(\frac{1}{\xi}u, \frac{1}{\xi}v, w\right)$ is the anisotropic modified velocity vector, $\nu = \frac{\mu}{\rho_0}$ is kinematic viscosity, $\xi = K_x/K_z$ is the mechanical anisotropy parameter and $\eta = \kappa_{Tx}/\kappa_{Tz}$ is the thermal anisotropy parameter. The parameter Va includes the thermal Prandtl number, Darcy number and the porosity δ of the medium and is given by

$$Va = \frac{\delta\,\mathrm{Pr}}{Da} \tag{18}$$

Now to eliminate the pressure term p from Eq.(15), we take the curl of it and obtain an equation in the form

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial w}{\partial t} - Q.P_m\frac{\partial}{\partial z}(\nabla\times\mathrm{H})\right] + \left(1+\lambda_2\frac{\partial}{\partial t}\right)\varsigma = \left(1+\lambda_1\frac{\partial}{\partial t}\right)\times\left[Ra\left(\frac{\partial T}{\partial y}\hat{i} - \frac{\partial T}{\partial z}j\right)\right] \tag{19}$$

where $\omega = \nabla\times q$ and $\varsigma = \nabla\times q_a$ denotes the vorticity vector and modified vorticity vector respectively and $q_a = \left(\frac{1}{\xi}u, \frac{1}{\xi}v, w\right)$. Applying curl on Eq.(19), we get the following equation

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial}{\partial t}(\nabla^2\mathrm{q}) - Q.P_m\nabla^2\left(\frac{\partial H}{\partial z}\right)\right] + \left(1+\lambda_2\frac{\partial}{\partial t}\right)C = \left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\hat{i}Ra\frac{\partial^2 T}{\partial x\partial z} + jRa\frac{\partial^2 T}{\partial y\partial z} - \hat{k}Ra\nabla_1^2 T\right] \tag{20}$$

where $C = (C_1, C_2, C_3)$ and

$$C_1 = \frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x} + \frac{\partial^2 w}{\partial x\partial z} - \left(\frac{\partial^2 v}{\partial y^2} + \frac{1}{\xi}\frac{\partial^2 u}{\partial z^2}\right),$$

$$C_2 = \frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 w}{\partial y\partial z} - \frac{1}{\xi}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right),$$

$$C_3 = -\left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w$$

3. Onset of Magnetoconvection

To obtain the information regarding the onset of magnetoconvection, We perform a linear stability analysis. For this, we neglect the nonlinear terms in the Eqs.(16), (17) and (20), and reduce the equations into the linear form. Then taking vertical component of the reduced equations, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial}{\partial t}(\nabla^2\mathrm{w}) - Q.P_m\nabla^2\left(\frac{\partial H_z}{\partial z}\right)\right] + \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right) = \left(1+\lambda_1\frac{\partial}{\partial t}\right)Ra\nabla_1^2 T \tag{21}$$

$$\left(\gamma\frac{\partial}{\partial t} - \eta\nabla_1^2 - \frac{\partial^2}{\partial z^2}\right)T = w \tag{22}$$

$$\left(\frac{\partial}{\partial t} - Pm\nabla^2\right)H_z = \frac{\partial w}{\partial z} \tag{23}$$

where w and H_z are the vertical components of velocity and magnetic field respectively.

Using Eqs.(21), (22) and Eq.(23), eliminating all variables except the vertical component of velocity, we get a single equation for w in the form

$$[\left(\gamma\frac{\partial}{\partial t} - \eta\nabla_1^2 - \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial}{\partial t} - P_m\nabla^2\right)\left\{\left(1+\lambda_1\frac{\partial}{\partial t}\right)\chi\frac{\partial}{\partial t}\nabla^2 + \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right\} - \left(1+\lambda_1\frac{\partial}{\partial t}\right)\times \left\{Ra\left(\frac{\partial}{\partial t} - P_m\nabla^2\right)\nabla_1^2 + QPm\left(\gamma\frac{\partial}{\partial t} - \eta\nabla_1^2 - \frac{\partial^2}{\partial z^2}\right)\nabla^2\frac{\partial^2}{\partial z^2}\right\}]\mathrm{w} = 0 \tag{24}$$

The boundaries are considered to be impermeable, isothermal and perfect electrically conducting, therefore we have the following conditions

$$w = T = H_z = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{25}$$

Here we use free-free boundaries for simplification of the problem. Normal mode technique is used to solve the above partial differential equation for W. For this we seek solution of the unknown field W in the form

$$w = W(\mathrm{z})\exp\left[\hat{i}(\mathrm{l\,x} + \mathrm{m\,y}) + \sigma\,\mathrm{t}\right] \tag{26}$$

where l and m are the horizontal wave numbers and σ is the growth rate, which in general a complex quantity given by $\sigma = \sigma_r + i\sigma_i$.. Substituting Eq.(26) in to Eq.(24), we obtain a single ordinary differential equation for $W(z)$ in the form

$$\begin{aligned}&[\{\gamma\sigma + \eta a^2 - D^2\}\{\sigma - Pm(D^2 - a^2)\}\times\\ &\left\{(1+\lambda_1\sigma)\frac{1}{Va}\sigma(D^2 - a^2) + (1+\lambda_2\sigma)\left(-a^2 + \frac{1}{\xi}D^2\right)\right\}\\ &+(1+\lambda_1\sigma)\begin{Bmatrix} a^2 Ra\{\sigma - Pm(D^2 - a^2)\} - \\ QPm(\gamma\sigma + \eta a^2 - D^2)(D^2 - a^2)\end{Bmatrix}]W(z) = 0\end{aligned} \tag{27}$$

where a is the horizontal wave number. The boundary conditions in terms of W are given by

$$W = \frac{d^2W}{dz^2} = \frac{d^4W}{dz^4} = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{28}$$

The above problem Eq.(27) and Eq.(28) can be regarded as an eigen value problem. The solutions of the boundary value problem are assumed to have the form $W(z) = A_n^{'}\sin(n\pi z)$, where $A_n^{'}$ denotes the amplitude which gives the minimum Darcy-Rayleigh number when $n = 1$, showing that $W(z) = A_1^{'}\sin(\pi z)$ is the eigen function for the marginal stability. Then the expression for the Darcy-Rayleigh number is obtained as

$$Ra = \frac{(\gamma\sigma + \eta a^2 + \pi^2)\left[(1+\lambda_1\sigma)\chi\sigma(\pi^2 + a^2) + (1+\lambda_2\sigma)\left(a^2 + \frac{1}{\xi}\pi^2\right)\right]}{a^2(1+\lambda_1\sigma)} + \frac{Q\,Pm(\gamma\sigma + \eta a^2 + \pi^2)(\pi^2 + a^2)\pi^2}{a^2\{\sigma + Pm(\pi^2 + a^2)\}} \tag{29}$$

3.1 Stationary State

For the occurrence of stationary convection, we consider that $\sigma = \sigma_r + i\sigma_i$. with the possibility that non zero σ would cause the overstability at the marginal state. Therefore at the marginal state we assume $\sigma = 0$ for stationary convection and obtain the expression forthe Darcy-Rayleigh number as

$$Ra^{st} = \frac{(\eta a^2 + \pi^2)\left[\left(a^2 + \frac{1}{\xi}\pi^2\right) + Q\pi^2\right]}{a^2} \tag{30}$$

For $\xi = \eta = 1$,we have

$$Ra^{st} = \frac{(a^2 + \pi^2)\left[(a^2 + \pi^2) + Q\pi^2\right]}{a^2} \tag{31}$$

Taking $\frac{\partial Ra^{st}}{\partial t} = 0$, we obtain critical wave number $a = a_c$ and the critical value of Darcy-Rayleigh number, as given below

$$a_c = \pi\left[(\xi\eta)^{-1}(1+\xi Q)\right]^{\frac{1}{4}} \text{ and } Ra_c^{st} = \pi^2\left[1 + \sqrt{\eta(\xi^{-1} + Q)}\right]^2 \tag{32}$$

For $\xi = \eta = 1$, we have

$$a_c = \pi(1+Q)^{\frac{1}{4}} \text{ and } Ra_c^{st} = \pi^2\left[1 + \sqrt{1+Q}\right]^2 \tag{33}$$

The above results (31) and (33) are similar to those obtained by Bhadauria and Sherani (2008b) for the onset of Darcy convection in magnetic fluid saturated porous medium. For the non-magnetoconvection $(Q = 0)$ and $\xi = \eta = 1$ (isotropic porous medium) we obtain

$$a_c = \pi \text{ and } Ra_c^{st} = 4\pi^2 \tag{34}$$

which are exactly same results as obtained by Lapwood (1948).

From the expression Eqs.(30) for Darcy-Rayleigh number for onset of stationary magnetoconvection, it is found that the critical Darcy-Rayleigh number and wave number are independent of the viscoelastic parameters and therefore same as magnetoconvection in an anisotropic porous medium saturated with Newtonian fluid.

3.2 Oscillatory state

We know that the oscillatory convection are possible only if some additional constraints like rotation, magnetic field, and salinity gradient, are present. For oscillatory convection at the marginal state, we must have $\sigma_r = 0$ and $\sigma_i \neq 0$. Now substituting $\sigma = i\sigma_i$ into the Eq.(29) and separating the real and imaginary part, we obtain

$$Ra^{OSC} = \frac{A^{'}\left[P_m^2(\pi^2 + a^2)^2 + \sigma_i^2\right]}{a^2(1+\lambda_1^2\sigma_i^2)\left[P_m^2(\pi^2 + a^2)^2 + \sigma_i^2\right]} + \frac{B'QP_m\pi^2(\pi^2 + a^2)(1+\lambda_1^2\sigma_i^2)}{a^2(1+\lambda_1^2\sigma_i^2)\left[P_m^2(\pi^2 + a^2)^2 + \sigma_i^2\right]} + i\sigma_i X \tag{35}$$

where X is given as

$$X = \frac{C'\left[P_m^2(\pi^2 + a^2)^2 + \sigma_i^2\right] + D'QP_m\pi^2(\pi^2 + a^2)(1+\lambda_1^2\sigma_i^2)}{a^2(1+\lambda_1^2\sigma_i^2)\left[P_m^2(\pi^2 + a^2)^2 + \sigma_i^2\right]} \tag{36}$$

the expressions for A', B', C' and D' are given in the Appendix. Since Oscillatory Darcy Rayleigh number Ra^{OSC} must be real therefore we must have $X = 0$. Thus we obtain a quadratic
Eq. in σ_i^2 in the form

$$K_1(\sigma_i)^2 + K_2(\sigma_i)^2 + K_3 = 0 \tag{37}$$

where the values of K_1, K_2 and K_3 are given in Appendix.

Since σ_i^2 has to be positive always, therefore for two positive roots of the Eq.(37) we must have $K_1 < 0$ and $K_2 > 0$ according to Descartes's rule of signs. If there are two positive real roots then minimum of σ_i^2 will gives the oscillatory neutral Darcy-Rayleigh number Ra^{OSC} corresponding to critical wave number a_c and critical value of σ_i^2. If there is no positive real root then no oscillatory motion is possible. However we found during our calculations that Eq.(37) has only one positive real root for some values of fixed parameters $\lambda_1, \lambda_2, \xi, \tau, \alpha, Q,\ Pm, \gamma$ and χ.

Further the corresponding value of the critical Darcy-Rayleigh number for the oscillatory mode is derived and is given in Eq.(35). It is found to be the function of mechanical anisotropy ξ, thermal anisotropy η, Darcy-Chandrasekhar number Q, relaxation time λ_1, the retardation time λ_2, a non dimensional number χ, Magnetic Prandtl number Pm, and of ratio of heat capacities γ. The graphically representation of these results is given in Section 5.

4. Weak Nonlinear Analysis

Now to extract the information about the rate of heat transfer and the convection amplitudes, we need to do a nonlinear analysis of the above problem. Therefore in this section, we will perform a weakly non-linear analysis and obtain some additional informations by considering a truncated representation of Fourier series for velocity, temperature and Magnetic field. This will be one step forward in understanding the non-linear mechanism of thermal convection. Here we have considered the case of two dimensional rolls, and thus made all physical quantities independent of y. We eliminate pressure term from Eq.(15) by operating $J.(\nabla\times)$ on it and introduce the stream function ψ such that $u = \partial\psi/\partial z,\ w = -\partial\psi/\partial x$ in the above resulting equation and in equation (16). Also we consider $H_x = \partial\phi/\partial z$ and $H_z = -\partial\phi/\partial x$ then we obtain

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\chi\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\psi - QPm\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi\right]+ \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi = -\left(1+\lambda_1\frac{\partial}{\partial t}\right)Ra\frac{\partial T}{\partial x} \tag{38}$$

$$\gamma\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z}+\frac{\partial\psi}{\partial x} = \left[\eta\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)+\frac{\partial^2}{\partial z^2}\right]T \tag{39}$$

$$\frac{\partial\phi}{\partial t}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial z}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z} = Pm\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi \tag{40}$$

To solve the above system, we use a minimal system of Fourier series by considering only two terms, thus we have expressions for stream function, temperature and Magnetic field as given by

$$\psi = A_1(\mathrm{t})\ \sin(\pi\,\mathrm{a}\,\mathrm{x})\sin(\pi\,\mathrm{z}) \tag{41}$$

$$\mathrm{T} = \mathrm{B}_1(\mathrm{t})\cos(\pi\,\mathrm{a}\,\mathrm{x})\sin(\pi\,\mathrm{z}) + C_1(\mathrm{t})\sin(2\pi\,\mathrm{z}) \tag{42}$$

$$\phi = \mathrm{D}_1(\mathrm{t})\sin(\pi\,\mathrm{a}\,\mathrm{x})\cos(\pi\,\mathrm{z}) + \mathrm{E}_1(\mathrm{t})\sin(2\pi\,\mathrm{a}\,\mathrm{x}) \tag{43}$$

Amplitudes $A_1(\mathrm{t}), \mathrm{B}_1(\mathrm{t}), \mathrm{C}_1(\mathrm{t}), \mathrm{D}_1(\mathrm{t})$ and $E_1(\mathrm{t})$ are functions of time and to be determined.

4.1 Steady Analysis

Here we take $\frac{\partial}{\partial t} = 0$ for the steady case, and assume that the amplitudes $A_1, \mathrm{B}_1, \mathrm{C}_1, \mathrm{D}_1$ and E_1 are constants. Substituting the above expressions for ψ, T and ϕ in Eqs.(38)-(40), and equating the coefficients of like terms of the resulting equations, we obtain

$$\pi^2\left(a^2+\frac{1}{\xi}\right)A_1 + \pi a\,Ra\,B_1 + Q\,Pm\,\pi^3\left(a^2+1\right)D_1 = 0 \tag{44}$$

$$\pi\,a\,A_1 + \pi^2\left[\eta\left(a^2+1\right)+1\right]B_1 + \pi^2 a\,A_1C_1 = 0 \tag{45}$$

$$\frac{\pi^2 a}{2}A_1B_1 - 4\pi^2\left(\eta+1\right)C_1 = 0 \tag{46}$$

$$\pi\,a\,A_1 + \pi^2\left(a^2+1\right)Pm\,D_1 + \pi^2 a\,A_1E_1 = 0 \tag{47}$$

$$\frac{\pi^2 a}{2}A_1D_1 + 4\pi^2a^2Pm\,E_1 = 0 \tag{48}$$

Further from the above equations, a single equation for the amplitude A_1 can be found as

$$\begin{aligned}&[\pi^2\left(a^2+\frac{1}{\xi}\right)\left\{(\eta+1)\left(\eta a^2+\eta+1\right)+a^2\left(\frac{A_1^2}{8}\right)\right\}\\&\left\{\left(a^2+1\right)P_m^2-\left(\frac{A_1^2}{8}\right)\right\}-\\&a^2Ra\left(\eta+1\right)\left\{\left(a^2+1\right)P_m^2-\left(\frac{A_1^2}{8}\right)\right\}-\\&QP_m^2\,\pi^3a\left(a^2+1\right)\left\{(\eta+1)\left(\eta a^2+\eta+1\right)+a^2\left(\frac{A_1^2}{8}\right)\right\}]A_1 = 0\end{aligned} \tag{49}$$

Since the solution $A_1 = 0$ corresponds to the pure conduction solution, therefore we put other part of the above equation as zero and obtain the amplitude equation as

$$x_1\left(\frac{A_1^2}{8}\right)^2 + x_2\left(\frac{A_1^2}{8}\right) + x_3 = 0 \tag{50}$$

$$x_1 = \pi^2 a^2 \left(a^2 + \frac{1}{\xi} \right)$$

$$x_2 = \pi^2 \left(a^2 + \frac{1}{\xi} \right)(\eta+1)\left(\eta a^2 + \eta + 1\right) + QP_m^2 \pi^3 a^2 \left(a^2+1\right) - P_m^2 a^2 \left(a^2+1\right) - a^2 Ra\, P_m^2 \left(a^2+1\right)(\eta+1)$$

And

$$x_3 = a^2\left(a^2+1\right)(\eta+1)\, Ra\, P_m^2 + QP_m^2 \pi^3 a\left(a^2+1\right)(\eta+1)\left(\eta a^2+\eta+1\right) - \pi^2\left(a^2+\frac{1}{\xi}\right)\left(a^2+1\right)P_m^2(\eta+1)\left(\eta a^2+\eta+1\right)$$

We note that the amplitude of stream function must be real, therefore we have to take positive sign in the root of Eq.(50). Once we determine the value of A_1, we can find the value of heat transfer. If $\overline{H}$ denotes the rate of heat transport per unit area, then we have

$$\overline{H} = -\kappa_{Tz} \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0} \tag{51}$$

where angular bracket represents the horizontal average. Also we have

$$T_{total} = T_0 - \Delta T \frac{z}{d} + T(\mathrm{x,z,t}) \tag{52}$$

Thus, we have

$$\overline{H} = \frac{\kappa_{Tz} \Delta T}{d}\left(1 - 2\pi C_1\right) \tag{53}$$

Further the expression for Nusselt number Nu can be given by

$$Nu = \frac{\overline{H}}{\kappa_{Tz}\, \Delta T/d} = 1 - 2\pi\, C_1 \tag{54}$$

Putting the value of C_1 in terms of A_1 we obtain

$$Nu = 1 + \frac{a^2\left(A_1^2/8\right)}{(\eta+1)\left(\eta a^2+\eta+1\right) + a^2\left(A_1^2/8\right)} \tag{55}$$

which is found to be the function of the parameters ξ, η, Ta, Ra and a. The corresponding results have been presented in the figures 2(a-d) and discussed in detail in section 5.

4.2 Non-steady Analysis

In this section we will perform the unsteady nonlinear analysis and investigate the transient behavior of Nusselt number with respect to time. Also we will study time dependent behavior of the stream function, temperature and magnetic field. For unsteady analysis of the problem, we solve Eqs.(38)-(40) with the help of Eqs.(41)-(43) and then equate the coefficients of like terms of resulting equations. We obtain the following set of nonlinear ordinary differential equations

$$\frac{dA_1(\mathrm{t})}{dt} = F_1(\mathrm{t}) \tag{56}$$

$$\frac{dB_1(\mathrm{t})}{dt} = -\frac{1}{\gamma}\left[\pi a A_1(\mathrm{t}) + \pi^2 K^2 B_1(\mathrm{t}) + \pi^2 a A_1(\mathrm{t}) \mathrm{C}_1(\mathrm{t})\right] \tag{57}$$

$$\frac{dC_1(\mathrm{t})}{dt} = -\frac{1}{\gamma}\left[\frac{\pi^2 a A_1(\mathrm{t}) \mathrm{B}_1(\mathrm{t})}{2} + 4\pi^2(\eta+1) C_1(\mathrm{t})\right] \tag{58}$$

$$\frac{dD_1(\mathrm{t})}{dt} = \pi a A_1(\mathrm{t}) - \pi^2\left(a^2+1\right) P_m D_1(\mathrm{t}) + \pi^2 a A_1(\mathrm{t}) \mathrm{E}_1(\mathrm{t}) \tag{59}$$

$$\frac{dE_1(\mathrm{t})}{dt} = -\left[\frac{\pi^2 a A_1(\mathrm{t}) \mathrm{D}_1(\mathrm{t})}{2} + 4\pi^2 a^2 P_m E_1(\mathrm{t})\right] \tag{60}$$

$$\frac{dF_1(\mathrm{t})}{dt} = \left[-\frac{\left(a^2+\frac{1}{\xi}\right)}{\lambda_1 \chi \left(a^2+1\right)} + \frac{Q P_m \pi^2 a}{\chi} + \frac{a^2 Ra}{\chi\gamma\left(a^2+1\right)}\right] A_1(\mathrm{t}) + \left[-\frac{a\, Ra}{\lambda_1 \chi \pi \left(a^2+1\right)} + \frac{\pi a\, Ra\, K^2}{\chi\gamma\left(a^2+1\right)}\right] B_1(\mathrm{t}) + \left[\frac{Q P_m \pi}{\lambda_1 \chi} - \frac{Q P_m^2 \pi^3 \left(a^2+1\right)}{\chi}\right] D_1(\mathrm{t}) + \frac{2\pi a^2 Ra}{\chi\gamma\left(a^2+1\right)} A_1(\mathrm{t}) \mathrm{C}_1(\mathrm{t}) + \frac{2Q P_m \pi^3 a}{\chi} A_1(\mathrm{t}) \mathrm{E}_1(\mathrm{t}) - \left[\frac{\lambda_2\left(a^2+\frac{1}{\xi}\right)}{\lambda_1\left(a^2+1\right)} + \frac{1}{\lambda_1}\right] F_1(\mathrm{t}) \tag{61}$$

The above system of simultaneous ordinary differential equations has been solved numerically using Runge-Kutta-Gill method ("Sastry, (1993)"). After determining the value of the amplitude functions $A_1(\mathrm{t}), \mathrm{B}_1(\mathrm{t}), \mathrm{C}_1(\mathrm{t}), \mathrm{D}_1(\mathrm{t})$ and $E_1(t)$, we evaluate the Nusselt number as a function of time.

5. Results and Discussions

Thermal instability in an anisotropic porous layer saturated with viscoelastic fluid has been investigated under a vertical magnetic field, using linear and nonlinear analyses. The linear stability analysis gives the conditions for stationary and oscillatory convection as presented in the Figs. 1 (a-h). In these Figs. 1(a-h), we draw neutral stability curves and depict the variation of the Darcy-Rayleigh number Ra with respect to the wave number a for the fixed values of the parameters $\xi = 0.7, \eta = 0.9,\ Q = 20, \lambda_1 = 0.8,\ \lambda_2 = 0.4, \gamma = 1.0, P_m = 0.4$ and a non-dimensional number $\chi = 1.6$, while varying one of the parameters.

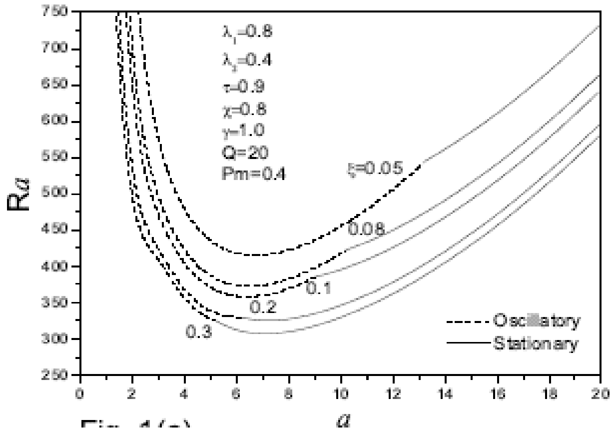

Fig. 1(a): Variation of Ra with wave number a.

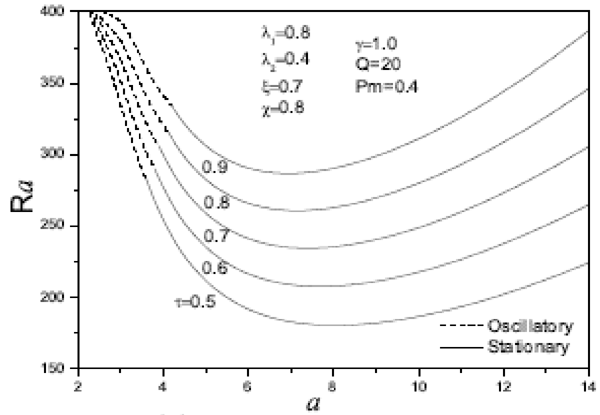

Fig. 1(b): Variation of Ra with wave number a.

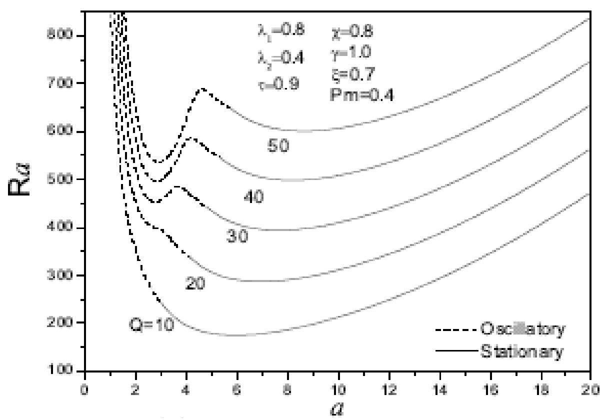

Fig. 1(c): Variation of Ra with wave number a.

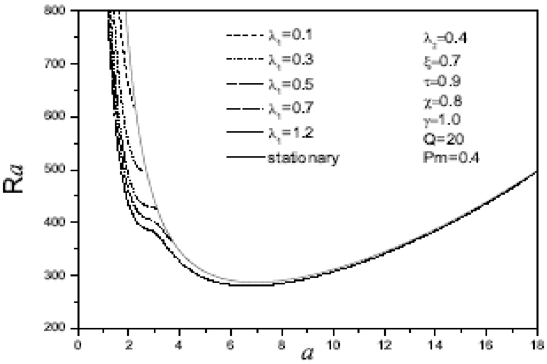

Fig. 1(d): Variation of Ra with wave number a.

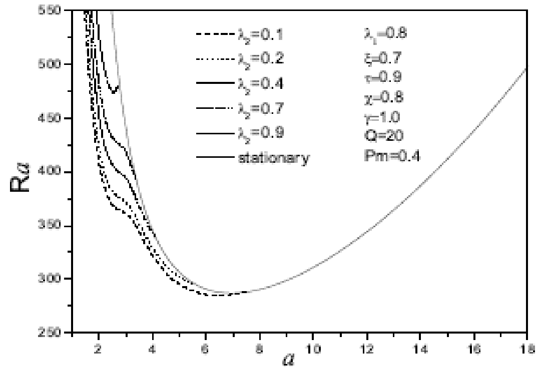

Fig. 1(e): Variation of Ra with wave number a.

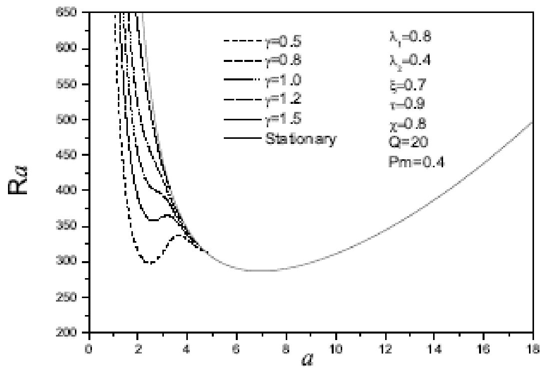

Fig. 1(f): Variation of Ra with wave number a.

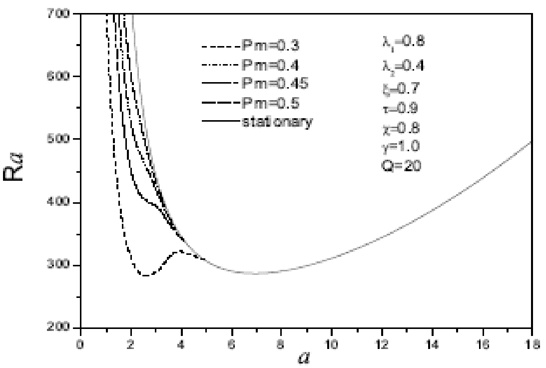

Fig. 1(g): Variation of Ra with wave number a.

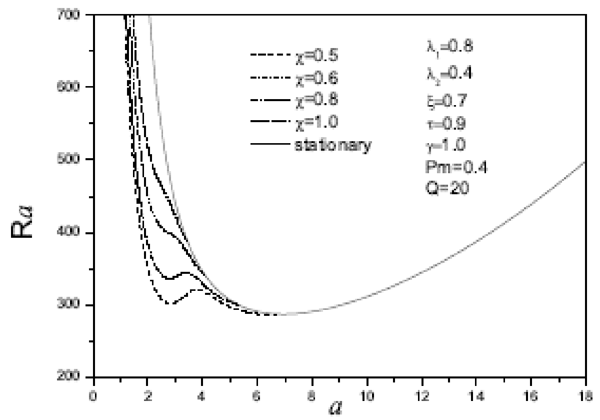

Fig. 1(h): Variation of Ra with wave number a.

We observed in most of the Figs. 1 (a-h) that when wave number a is small oscillatory convection sets in earlier than the stationary convection, however for intermediate and large values of the wave number a, stationary convection prevails. These figures give the criteria for thermal instability in terms of the critical Darcy-Rayleigh number Ra_c. If the value of the Darcy-Rayleigh number is below Ra_c the system will remain stable, however above this value the system become unstable and the onset of convection will occur. Further for instability to set in as over stability the condition $\sigma_i^2 > 0$ is only a necessary condition but not sufficient. For this to happen we must have the condition that $Ra^{osc} \le Ra^{st}$, which is observed in most of the Figs. 1 (a-h), for small values of a. Further from Fig. 1(a-c) we clearly see the points of intersection of Ra^{OSC} and Ra^{st} at a particular value of a^*. If a is less than a^* then oscillatory convection otherwise convection will set in as stationary. Also from the Figs. 1(d-h) we can clearly see the bifurcation points where the curves corresponding to the oscillatory convection branch off the stationary convection curves.

From Figs. 1(b) and 1(c) we observed respectively, that on increasing η and Q the value of Ra increases, however it decreases on increasing mechanical anisotropy ξ (Fig. 1(a)). Thus increments in η and Q make the system stabilized, however an increment in ξ, makes the system destabilized. From Fig. 1(d), we find that the instability sets in from left to bifurcation point as over stability and on decreasing the value of the relaxation time λ_1, the critical value of Ra^{OSC} increases and thus system becomes more stabilized. However on further decreasing the value of λ_1, the over stability shifts towards right and the critical value of Ra^{OSC} further decreases. In Figs. 1(e-h), we find qualitatively similar results to Fig. 1(d) as here also the instability sets in as over stability form left to the bifurcation point. However Ra^{OSC} increases on increasing the values of the parameters λ_2, γ, P_m and χ, thus stabilizing the system.

Weakly nonlinear stability analysis of the problem is carried out using truncated representation of the Fourier series and the informations regarding the rate of heat transfer across the porous layer are obtained. The effect of time on Nusselt number is also investigated by considering nonlinear, unsteady problem. At the end we obtained some graphs for the steam lines, isotherms and the magnetic stream lines.

First we investigate the nonlinear steady problem, and present the results in Figs. 2(a-d). In these figures, we depict the variation in Nusselt number Nu with respect to the Darcy Rayleigh number Ra for different values of the mechanical anisotropy ξ, thermal anisotropy η, Darcy-Chandrasekhar number Q and the magnetic Prandtl number P_m respectively. From these figures, we find that the value of the Nusselt number Nu increases on increasing the value of Ra, which shows that heat transfer across the porous layer increases on increasing the value of Ra.

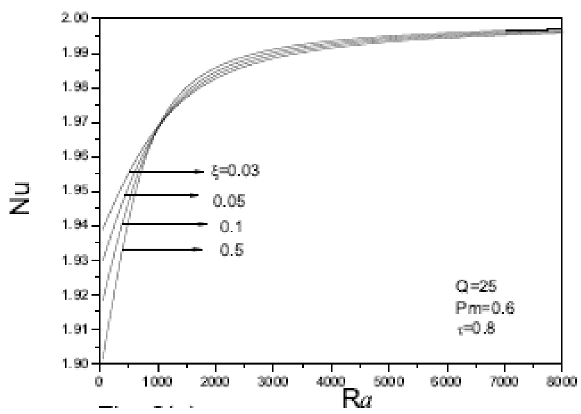

Fig. 2(a): Variation of Nu with wave number Ra

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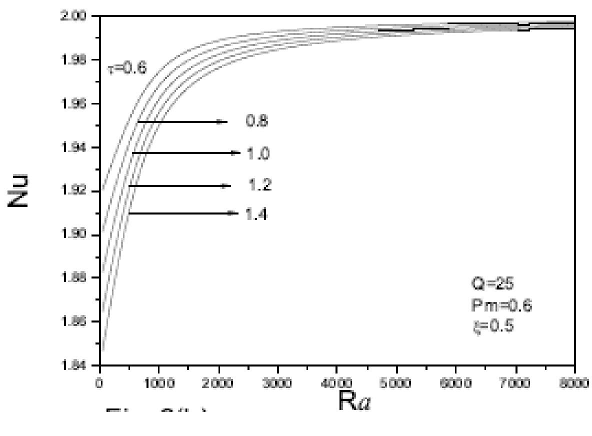


Fig. 2(b): Variation of Nu with wave number Ra.

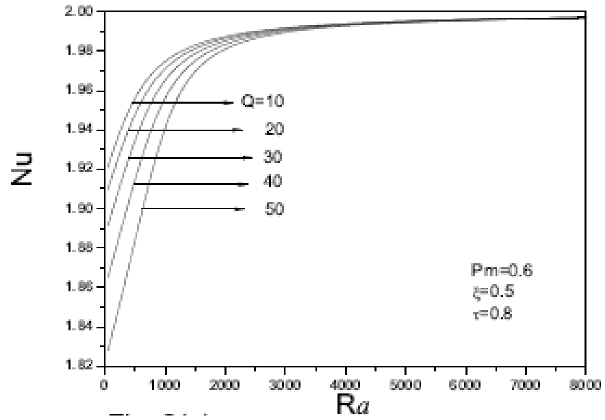


Fig. 2(c): Variation of Nu with wave number Ra.

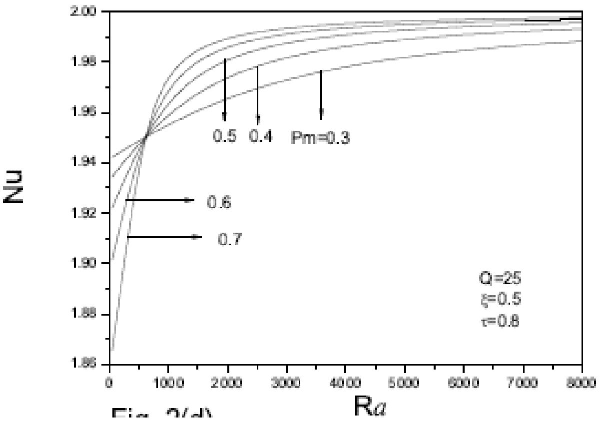


Fig. 2(d): Variation of Nu with wave number Ra.

However when the value of Ra is sufficiently large, the value of Nu becomes almost constant i.e. beyond a certain value of Ra, the heat transfer across the porous layer remains constant. In Fig. 2(a), we exhibit the effect of ξ on heat transfer. From the figure we see that when Ra < 1150, the value of Nu decreases on increasing ξ, at about Ra = 1150 the value of Nu becomes almost same and when Ra > 1150, Nu increases with increasing ξ. This shows that the effect of mechanical anisotropy ξ is to suppress the convection initially and then advance it. In Figs. 2(b) and 2(c) we display the effects of thermal anisotropy η and Q on the Nusselt number Nu. From the figures, we observe that on increasing the value of η and Q, the value of Nu decreases, thus decreasing the rate of heat transfer across the porous medium. Further in Fig. 2(d), we obtain qualitatively similar result as shown in Fig. 2(a), for different values of magnetic Prandtl number P_m. Here we discuss the transient behavior of the system by solving the autonomous system of ordinary differential Eqs. (56)-(61) numerically, using Runge −Kutta−Gill Method, and calculate Nusselt number Nu as function of time t. The Figs. 3 (a-i) depict the response of the time t corresponding to the Nusselt number Nu to variation in one of the parameters, while the others are held fixed at their respective values;

$$\lambda_1 = 0.2, \lambda_2 = 1.0, \gamma = 0.6, P_m = 0.4$$
$$\xi = 0.2, \eta = 0.6, Q = 20, \chi = 1.6 \text{ and } Ra = 200$$

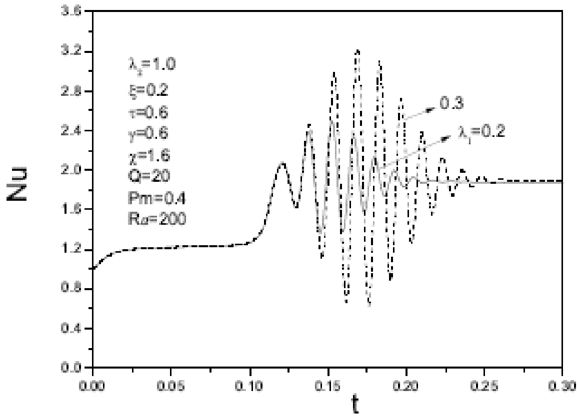


Fig. 3(a): Variation of Nu with wave number t.

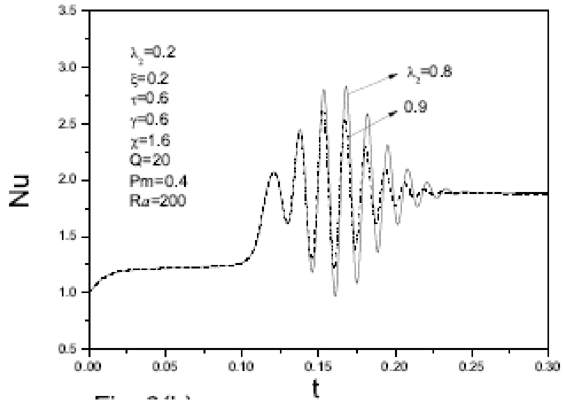


Fig. 3(b): Variation of Nu with wave number t.

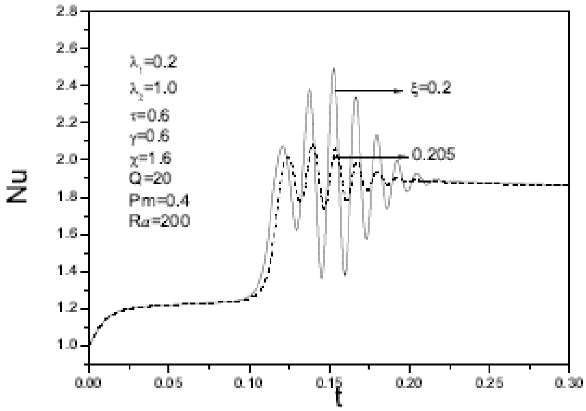


Fig. 3(c): Variation of Nu with wave number t.

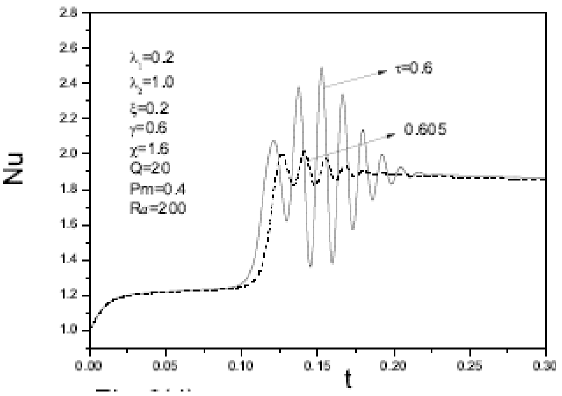


Fig. 3(d): Variation of Nu with wave number t .

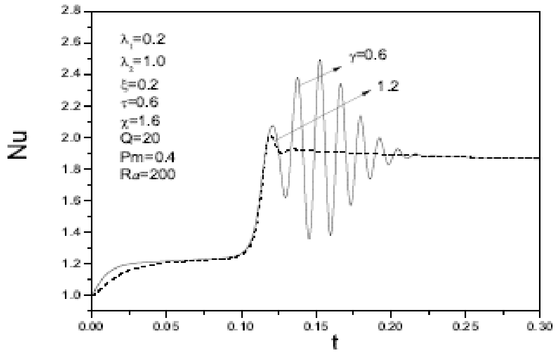


Fig. 3(e): Variation of Nu with wave number t .

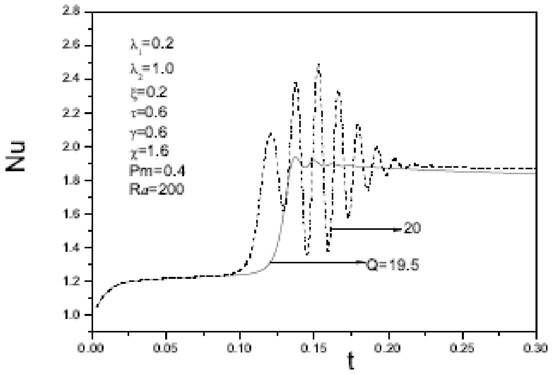


Fig. 3(f): Variation of Nu with wave number t .

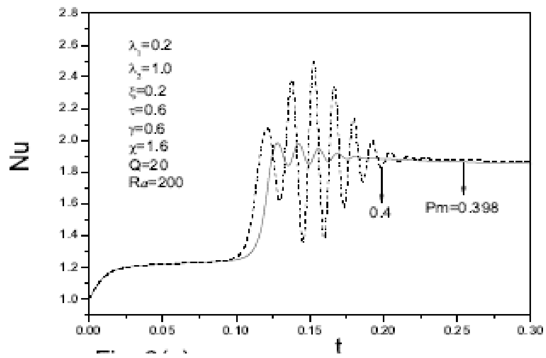


Fig. 3(g): Variation of Nu with wave number t .

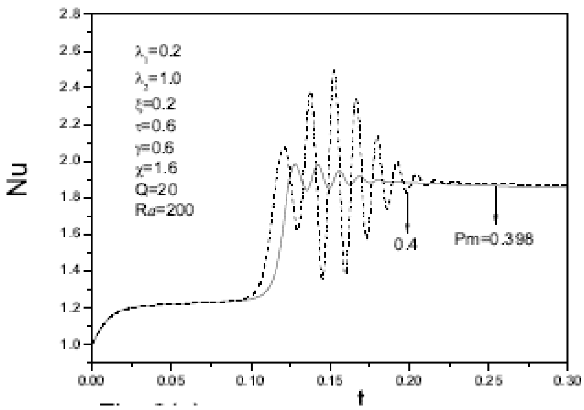


Fig. 3(g): Variation of Nu with wave number t .

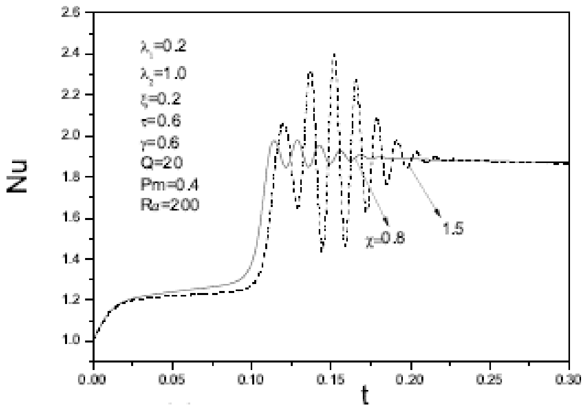


Fig. 3(h): Variation of Nu with wave number t .

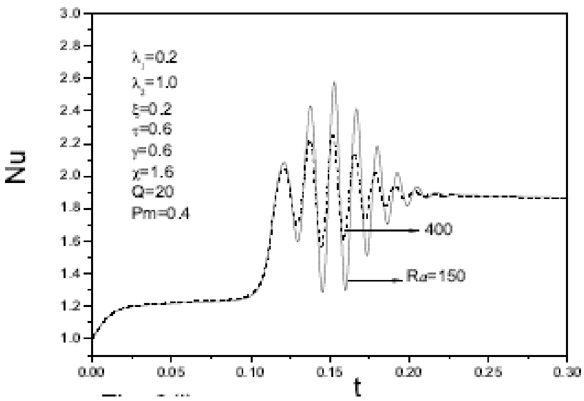
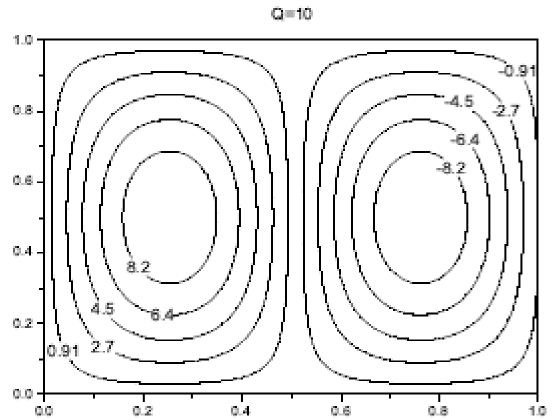


Fig. 3(i): Variation of Nu with wave number t .

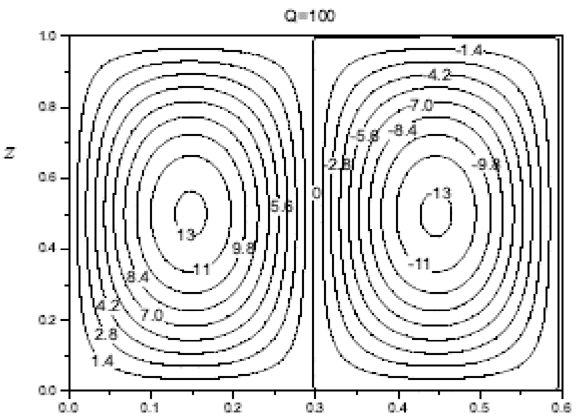
It is found from the figures that initially the value of the Nusselt number Nu is 1 at t = 0. It increases and oscillates at intermediate values of time t, and then becomes almost constant and approaches the steady state value at very large value of time t. The effects of various parameters on the Nusselt number Nu for unsteady case are found to be the same as that for steady state case.

In Figs. 4(a-b), we draw streamlines for $Q=10$ and $Q=100$ at $P_m=0.3, \xi=0.5, \eta=0.9$. From these figures, we see that stream lines are equally divided.



$x \quad Pm = 0.3, \xi = 0.5, \tau = 0.8$

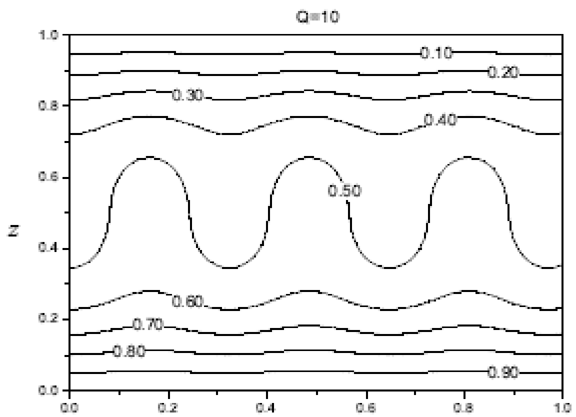
Fig 4(a): Stream lines for $Q = 10$.



$x \quad Pm = 0.3, \xi = 0.5, \tau = 0.8$

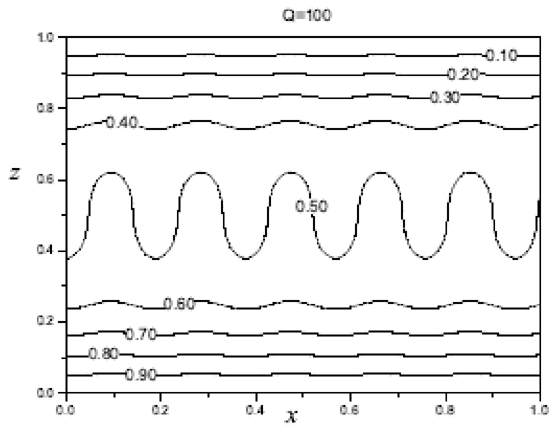
Fig 4(b): Stream lines for $Q = 100$.

The effect of increasing the Darcy-Chandrasekhar number Q is to decrease the wavelength of the cells, thereby contracting the cells. Isotherms are drawn in Figs. 5 (a-b) for $Q = 10$ and $Q = 100$.



$x \quad Pm = 0.3, \xi = 0.5, \tau = 0.8$

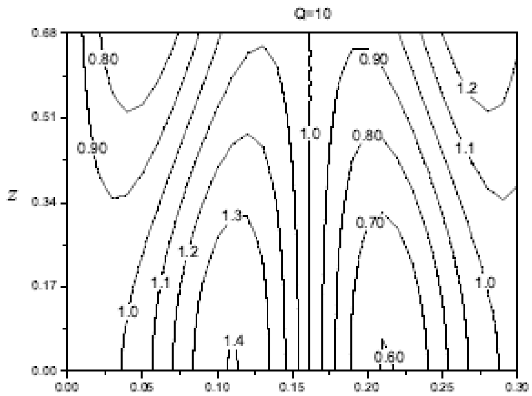
Fig 5(a): Isotherms for $Q = 10$.



$Pm = 0.3, \xi = 0.5, \tau = 0.8$

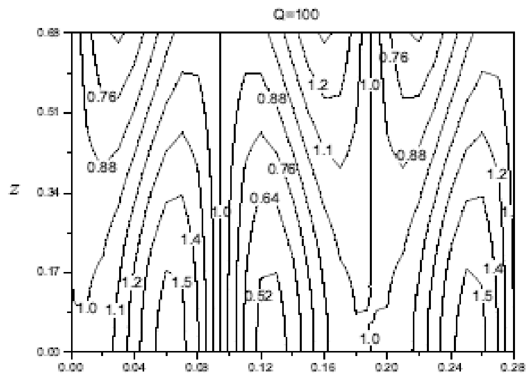
Fig 5(b): Isotherms for $Q = 100$.

We observed that isotherms are almost horizontal at the boundaries and oscillatory in the middle of the porous layer, thus showing conductive nature at the boundaries and convective behavior in the middle of the system. The isotherms become more oscillatory in nature on increasing the value of Q. Magnetic stream functions are drawn in Figs. 6(a-b).



$x \quad Pm = 0.3, \xi = 0.5, \tau = 0.8$

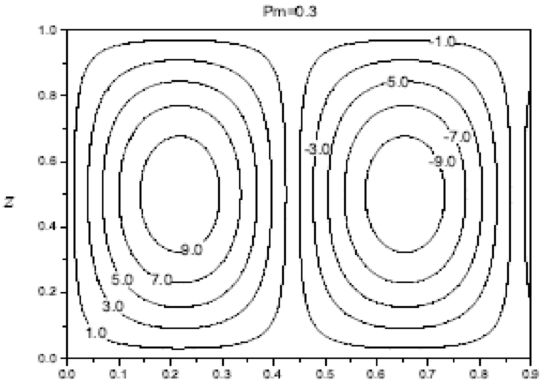
Fig. 6(a): Magnetic stream function for $Q = 10$.



$x \quad Pm = 0.3, \xi = 0.5, \tau = 0.8$

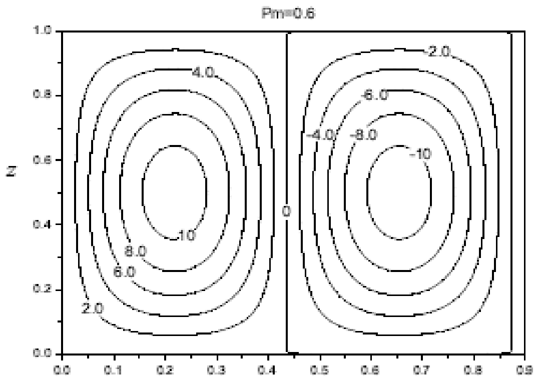
Fig. 6(b): Magnetic stream function for $Q = 100$.

Here also we observed that the effect of increase in the magnitude of the magnetic field is to contract the cells, thereby reducing the wavelength of the cells. Further in Figs. 7-9, which are drawn at P m = 0.3 and P m = 0.6, respectively for streamlines, isotherms and magnetic streamlines, we find qualitatively similar results to Figs. 4-6. Also we calculated the results for velocity streamlines, isotherms and magnetic streamlines in unsteady case and found that when t is very large, these results approach those which are presented in the figures 4-9.



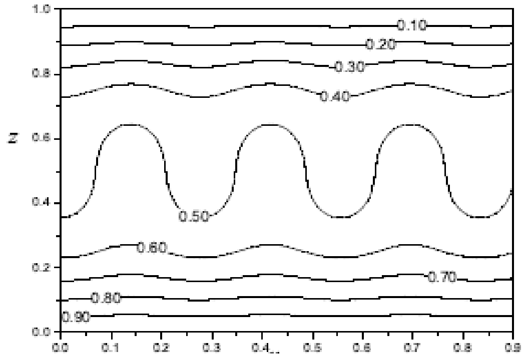
$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 7(a): Stream lines for $Pm=0.3$.



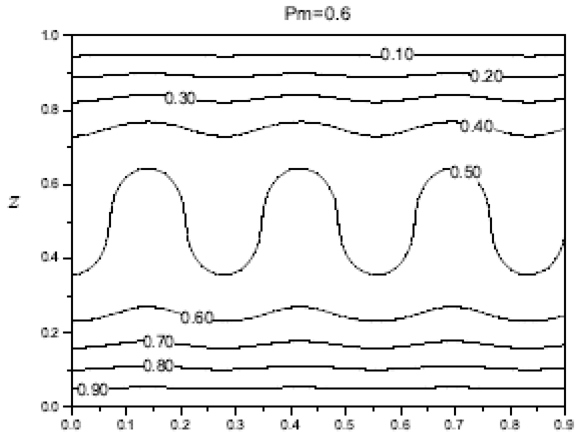
$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 7(b): Stream lines for $Pm=0.6$.



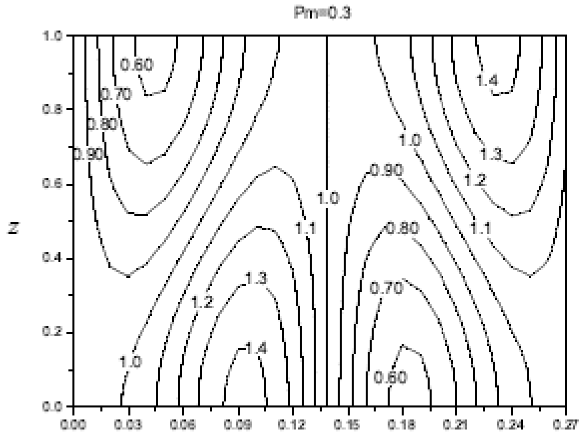
$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 8(a): Isotherms for $Pm=0.3$.



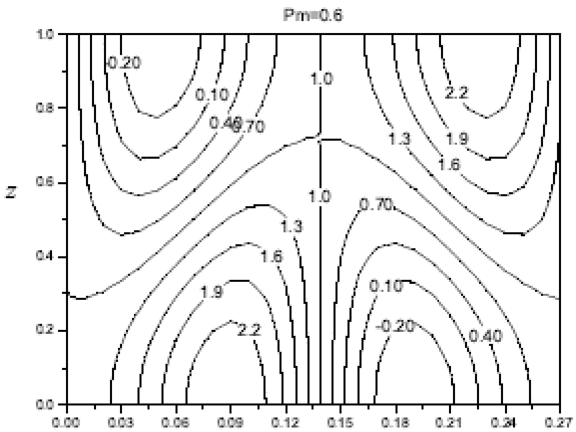
$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 8(b): Isotherms for $Pm=0.6$.



$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 9(a): Magnetic Stream functions for $Pm=0.3$.



$x \quad Q=20, \xi=0.5, \tau=0.8$

Fig. 9(b): Magnetic Stream functions for $Pm=0.6$.

Conclusion

The effect of magnetic field on the onset of convection in an anisotropic horizontal porous layer saturated with viscoelastic fluid has been investigated. The problem has been solved analytically, performing linear and weakly nonlinear analyses. The results have been obtained for steady and non-steady case. The following points have been observed:

1. We obtain both stationary as well as oscillatory convection, depending on the values of the parameters.
2. In linear analysis, the effect of increasing the values of ξ is found to decrease the value of Ra thus advancing the onset of convection.

3. The effect of increasing the values of η and Q is found to increase the value of Ra thus delaying the onset of convection.

4. The critical value of Ra^{osc} increases on decreasing the value of the relaxation time λ_1 and on increasing the value of retardation time λ_2.

5. For nonlinear, steady motion, the effects of ξ, η, Q and P_m are found to suppress the convective flow since the value of Nu decreases on increasing the values of ξ, η, Q and P_m.

6. In nonlinear unsteady motion the effects of increasing ξ, η and Q are to reduce the heat transfer, thus suppressing the convection, however the effects of λ_1 and λ_2 are found to enhance the heat transfer. Furthermore the value of Nu approaches the steady value for large value of t.

7. Finally we find that the effect of increasing Q and P_m is to decrease the wavelength of the cells thereby contracting the cells. Further isotherms are found to be more oscillatory in nature on increasing Q and P_m.

Appendix A

$$A' = \left[\left(\eta a^2 + \pi^2\right) + \gamma\lambda_1\sigma_i^2\right]\left[\left(a^2 + \frac{1}{\xi}\pi^2\right) - \chi\lambda_1\left(\pi^2 + a^2\right)\sigma_i^2\right] - \left[\gamma - \lambda_1\left(\eta a^2 + \pi^2\right)\right]\left[\lambda_2\left(a^2 + \frac{1}{\xi}\pi^2\right) + \chi\left(a^2 + \pi^2\right)\right]\sigma_i^2 \tag{A1}$$

$$B' = P_m\left(\eta a^2 + \pi^2\right)\left(a^2 + \pi^2\right) + \gamma\,\sigma_i^2 \tag{A2}$$

$$C' = \left[\left(\eta a^2 + \pi^2\right) + \gamma\lambda_1\sigma_i^2\right]\left[\lambda_2\left(a^2 + \frac{1}{\xi}\pi^2\right) + \chi\left(a^2 + \pi^2\right)\right] + \left[\gamma - \lambda_1\left(\eta a^2 + \pi^2\right)\right]\left[\left(a^2 + \frac{1}{\xi}\pi^2\right) - \chi\lambda_1\left(\pi^2 + a^2\right)\sigma_i^2\right] \tag{A3}$$

$$D' = \gamma P_m\left(a^2 + \pi^2\right) - \left(\eta a^2 + \pi^2\right) \tag{A4}$$

$$K_1 = \lambda_1^3\lambda_2\pi^2 Ta\xi + \lambda_2^3\lambda_1\left(a^2 + \frac{1}{\xi}\pi^2\right) \tag{A5}$$

$$K_2 = 2\left[\pi^2 Ta\xi\,\lambda_1 + \left(a^2 + \frac{1}{\xi}\pi^2\right)\lambda_2\right]\left[\left(\lambda_1 + \lambda_2\right) - \lambda_1\lambda_2\left(\tau a^2 + \pi^2\right)\right] - \left[1 - \left(\tau a^2 + \pi^2\right)\left(\lambda_1 + \lambda_2\right)\right]\left[\pi^2 Ta\xi\,\lambda_1^2 + \left(a^2 + \frac{1}{\xi}\pi^2\right)\lambda_2^2\right] - \lambda_1\lambda_2\left[\pi^2 Ta\xi + \left(a^2 + \frac{1}{\xi}\pi^2\right)\right] \tag{A6}$$

$$K_3 = 2\left(\tau a^2 + \pi^2\right)\left[\pi^2 Ta\xi\,\lambda_1 + \left(a^2 + \frac{1}{\xi}\pi^2\right)\lambda_2\right] + \left[1 - \left(\tau a^2 + \pi^2\right)\left(\lambda_1 + \lambda_2\right)\right]\left[\pi^2 Ta\xi + \left(a^2 + \frac{1}{\xi}\pi^2\right)\right] \tag{A7}$$

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