

Theoretical and Numerical Analysis of Half Car Vehicle Dynamic Model Subjected to Different Road Profiles with Wheel Base Delay and Nonlinear Parameters

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Abstract

Suspension Model design is a challenging task for the automobile designers in view of multiple control parameters, complex (often conflicting) objectives and stochastic disturbances. The roles of a suspension Model are to support the vehicle weight, to isolate the vehicle body from road disturbances, and to maintain the traction force between the tire and the road surface. The purpose of suspension Model is to improve the ride comfort, road handling and stability of vehicles. For proper designing of suspension Model, nonlinearities in suspension parameters must be considered. In this paper, nonlinearities of tire spring and damper are considered while preparing half car model. The current article is simulated and analyzed for the handling and ride performance of a vehicle with passive suspension Model of Half Car Model with Four Degree of Freedom. Since, the equations of the Model cannot be solved mathematically; MATLAB/Simulink has developed a scheme that allows analyzing the behavior of the suspension Model for different road profiles.

Keywords: Vehicle Dynamic System, Half Car Model, MATLAB /Simulink, Nonlinear Passive Suspension Model, Wheel Base Delay.

1. Introduction

For vehicle suspension design, it is always challenging to maintain simultaneously a high standard of ride, handling, and body attitude control under all driving conditions. The problems stem from the wide range of operating conditions created by varying road conditions, vehicle speed, and load. In general, during cornering, braking, and bumping, a high stiffness and damping is needed to provide good handling properties, and to satisfy workspace limitations of the suspension Model. However, when a vehicle runs on a low roughness road, a suspension Model with low stiffness and damping is needed for good ride comfort. A good suspension Model should provide good vibration isolation, i.e. small acceleration of the body mass, and a small "rattle space", which is the maximal allowable relative displacement between the vehicle body and various suspension components [1]. The goal is to maintain the suspension travel within the rattle space and to minimize car-body rate-of-change of acceleration.

Ride comfort is one of the most critical factors to evaluate the automobile performance and has been an interesting topic for researchers for many years. Automobile designers give an abundant attention to the isolation of vibrations in the car, in order to provide a comfortable ride for the passengers.

2. The mathematical model of half car suspension model

The purpose of mathematical modeling is to obtain a state space representation of the vehicle model. Though the quarter car model is simple and widely used for dynamic performance analysis, it fails to capture the more realistic results of actual behavior of the vehicle, so in this work half-car vehicle model shown in Fig.1, which captures important characteristics of full car model, is used for analysis.

The Model shown in Fig.1 is an half car Model where m_s - Sprung Mass, m_{uf} - Front Unsprung Mass, m_{ur} - Rear Unsprung Mass, k_f - Front Suspension Stiffness, k_r - Rear Suspension Stiffness, c_f - Front Suspension Damping Coefficient, c_r - Rear Suspension Damping Coefficient k_t - Tyre (195/65R15) Stiffness, c_t -Tyre (195/65R15) Damping Coefficient, r - Radius of Gyration, b -Wheel Base. The model comprises of sprung mass and two unsprung masses. The x_s denotes the vertical displacement (bounce) at the center of gravity and θ is the pitch of the sprung mass. The front and rear displacements of the unsprung masses are denoted by x_{uf} and x_{ur} . In this model, the disturbances q_f and q_r are caused by road irregularities.

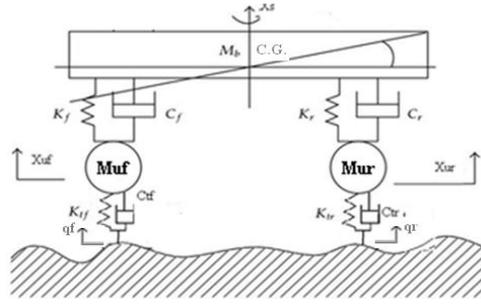


Fig.1: 4 DoF Half Car Vehicle Model

By using Newton’s second law, the equations of motion for linear half car passive suspension Model can be written as

$$m_s \ddot{x}_s + k_f(x_s - l_f \theta - x_{uf}) + c_f(\dot{x}_s + l_f \dot{\theta} - \dot{x}_{uf}) + k_r(x_s + l_r \theta - x_{ur}) + c_r(\dot{x}_s - l_r \dot{\theta} - \dot{x}_{ur}) = 0 \tag{1}$$

$$I \ddot{\theta} + k_f(x_s - l_f \theta - x_{uf})l_f - c_f(\dot{x}_s + l_f \dot{\theta} - \dot{x}_{uf})l_f + k_r(x_s + l_r \theta - x_{ur})l_r + c_r(\dot{x}_s - l_r \dot{\theta} - \dot{x}_{ur})l_r = 0 \tag{2}$$

$$m_{uf} \ddot{x}_{uf} + k_{tf}(x_{uf} - q) + c_{tf}(\dot{x}_{uf} - \dot{q}) + k_f(-x_s + l_f \theta + x_{uf}) + c_f(-\dot{x}_s + l_f \dot{\theta} + \dot{x}_{uf}) = 0 \tag{3}$$

$$m_{ur} \ddot{x}_{ur} + k_{tr}(x_{ur} - q) + c_{tr}(\dot{x}_{ur} - \dot{q}) + k_r(-x_s - l_r \theta + x_{ur}) + c_r(-\dot{x}_s - l_r \dot{\theta} + \dot{x}_{ur}) = 0 \tag{4}$$

The equations of motion by considering tire spring nonlinearities for nonlinear tire spring passive system are

$$m_{uf} \ddot{x}_{uf} + (k_{g0} + k_{g1}v + k_{g2}v^2 + k_{g3}v^3)(x_{uf} - q) + c_{tf}(\dot{x}_{uf} - \dot{q}) + k_f(-x_s + l_f \theta + x_{uf}) + c_f(-\dot{x}_s + l_f \dot{\theta} + \dot{x}_{uf}) = 0 \tag{5}$$

$$m_{ur} \ddot{x}_{ur} + (k_{r0} + k_{r1}v + k_{r2}v^2 + k_{r3}v^3)(x_{ur} - q) + c_{tr}(\dot{x}_{ur} - \dot{q}) + k_r(-x_s - l_r \theta + x_{ur}) + c_r(-\dot{x}_s - l_r \dot{\theta} + \dot{x}_{ur}) = 0 \tag{6}$$

The equations of motion for nonlinear tire damper passive suspension system are

$$m_{uf} \ddot{x}_{uf} + [k_{tf}(x_{uf} - q)] + [(c_{t0} + c_{t1}v + c_{t2}v^2 + c_{t3}v^3)_f(\dot{x}_{uf} - \dot{q})] + [k_{sf}(-x_s + l_f \theta + x_{uf})] + [c_{sf}(-\dot{x}_s + l_f \dot{\theta} + \dot{x}_{uf})] = 0 \tag{7}$$

$$m_{ur} \ddot{x}_{ur} + [k_{tr}(x_{ur} - q)] + [(c_{t0} + c_{t1}v + c_{t2}v^2 + c_{t3}v^3)_r(\dot{x}_{ur} - \dot{q})] + [k_{sr}(-x_s - l_r \theta + x_{ur})] + [c_{sr}(-\dot{x}_s - l_r \dot{\theta} + \dot{x}_{ur})] = 0 \tag{8}$$

In order to analyze the behavior of the half car suspension, model is simulated in MATLAB/Simulink

The input parameters are as follows,

Table 1: Suspension Parameters for Hyundai Elantra 1992 Half Car Model

Sprung Mass (m_s)	515.45 Kg
Front Unsprung Mass (m_{uf})	23.61 Kg
Rear Unsprung Mass (m_{ur})	28 Kg
Front Suspension Stiffness (k_f)	12.394kN/m
Rear Suspension Stiffness (k_r)	14.662kN/m
Front Suspension Damping Coefficient (c_f)	1.3854 kN-sec/m
Rear Suspension Damping Coefficient (c_r)	1.6352kN-Sec/m
Tyre (195/65R15) Stiffness (k_t)	181.81888kN/m
Tyre (195/65R15) Damping Coefficient (c_t)	0.0138kN-sec/m
Radius of Gyration (r)	1.55 m
Wheel Base (b)	2.5 m

Vehicle is assumed to be traveling over a road with velocity of 40km/hr, during this travel the excitation frequency is calculated as

$$f = \frac{2\pi V}{\lambda} = \frac{2 \times \pi \times 40 \times 1000}{6 \times 3600} = 11.6 \text{ rad/sec} = 1.85\text{Hz}$$

3. Nonlinearity in Tire

Vehicle tire is component which primarily acts as kind of suspension Model, amongst other functions of tire, absorption of shocks from road and avoids these shocks as much as possible from transferring to main suspension Model. Tire is mainly composed of metal strings and rubber because of this it has nonlinear stiffness and damping property. Hence nonlinearities in those stiffness and damper are required to be considered.

3.1 Nonlinearity in tire spring

The tire characteristic is not constant; it depends on the inflation pressure and speed of tires. The non-linear effects can be included in tire spring force f_t with respect to speed. The tire stiffness is modeled as shown in Fig.2.

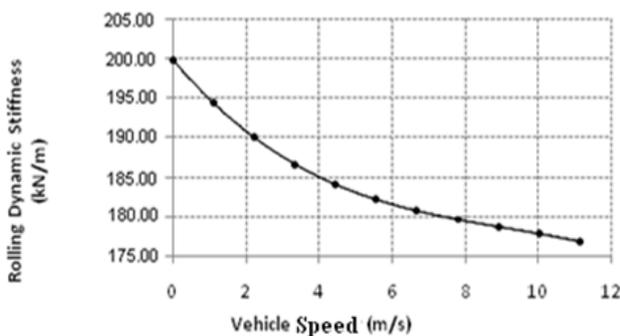


Fig.2 Non-linear Rolling Dynamic Stiffness Property of Hyundai Elantra 1992 Model Tire

The tire spring force f_{kt} is modeled as third order polynomial function as,

$$f_{kt} = (k_{t0} + k_{t1}v + k_{t2}v^2 + k_{t3}v^3)(q - x_u) \tag{9}$$

The co-efficients of these equations are obtained by fitting the experimental data the values of these co-efficients are

Table 2 Nonlinearity in Tire Spring

Sr.No	Parameter	Value
1	k_{tf0} and k_{tr0}	199987.71N/m
2	k_{tf1} and k_{tr1}	-5501.6 12 N/m ²
3	k_{tf2} and k_{tr2}	520.3612 N/m ³
4	k_{tf3} and k_{tr3}	-19.12 N/m ⁴

3.2 Nonlinearity in Tire Damper

Hysteresis of tire materials causes damping in a pneumatic tire. Value of damping coefficient depends on the design and construction of the tire, as well as operating conditions. The damping of pneumatic tires made of synthetic rubber compounds is considerable less than that provided by a shock absorber. From the measured data the tire damping coefficient is modeled as shown in Fig.3.

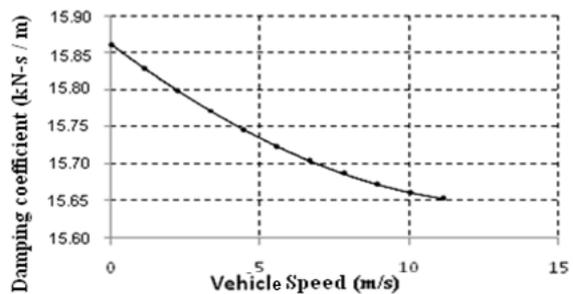


Fig.3 Non-linear Damping Coefficient Property of Hyundai Elantra 1992 Model Tire

From above graph the tire damping force f_{dt} is modeled as third order polynomial function as,

$$f_{dt} = (c_{t0} + c_{t1}v + c_{t2}v^2 + c_{t3}v^3)(\dot{q} - \ddot{x}_u) \tag{10}$$

The co-efficient of these equations are obtained by fitting the experimental data the values of these co-efficients are

Table 3 Nonlinearity in Tire Damper

Sr.No	Parameter	Value
1	c_{tr0} and c_{tr0}	15023 N-s/m
2	c_{tf1} and c_{tr1}	-4163 N-s ² /m ²
3	c_{tf2} and c_{tr2}	630 N-s ³ /m ³
4	c_{tf3} and c_{tr3}	-33.49 N-s ⁴ /m ⁴

4. Matlab Analysis Results

The MATLAB/Simulink model is prepared and the sprung mass displacement and pitch for different road profiles were obtained in time domain. The sprung mass displacement and pitch for linear and non-linear passive suspension Model for different road profiles are as below,

4.1 Bumpy Road (Sinusoidal Input) Profile

A single bump road input, y is described by (Jung-Shan Lin 1997), is used to simulate the road to verify the developed control Model. The road input is described by the equation (11) is shown in Fig.4.

$$y = a(1 - \cos \omega t) \text{ for } 0.4 < t < 0.9 \tag{11}$$

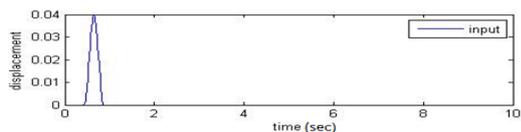


Fig.4 Bumpy Road Input

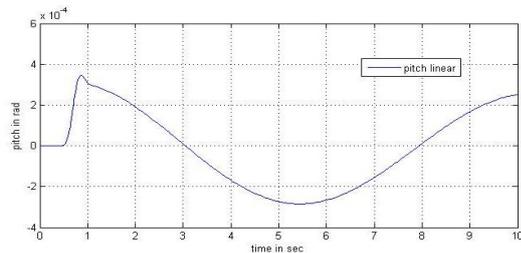


Fig.5 Pitch of Sprung Mass for Linear and Non-Linear Half Car Model

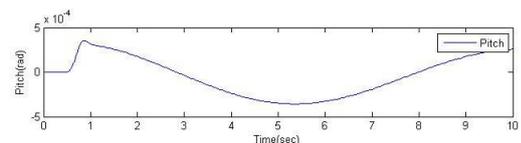


Fig.6 Sprung Mass Displacement (Xs) For Linear And Non-Linear Half Car Model

From the results obtained it is observed that the Pitch of Sprung Mass for Bumpy road is less than Rectangular and Step Input and Sprung Mass Displacement is less than Rectangular Input.

4.2 Rectangular Input

The Rectangular Pulse is represented by equation (12)

$$y = -0.04 \text{ for } 0.42 < t < 0.88, 0.00 \text{ for otherwise} \quad (12)$$

The road input is described by Eq.(12) is shown in Fig.7.

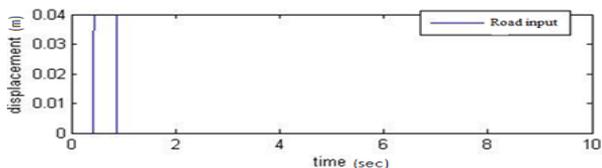


Fig. 7 Rectangular Input

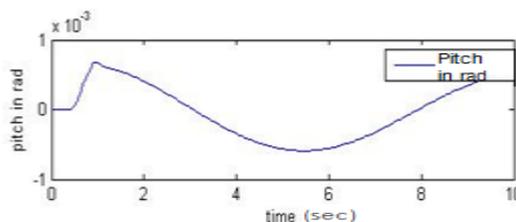


Fig.8 Pitch of Sprung Mass for (θ) Linear and Non-Linear Half Car Model

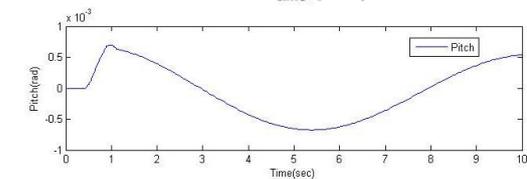
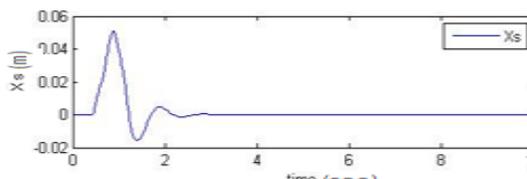


Fig.9 Sprung Mass Displacement (Xs) for Linear and Non-Linear Half Car Model



From the results obtained it is observed that the Pitch of Sprung Mass for Rectangular Input is larger than Bumpy and Step Input and Sprung Mass Displacement for Rectangular Input is larger than Bumpy and Step Input.

4.3 Step Input

The Step excitation is represented by equation (13)

$$y = -0.04 \text{ for } 0.42 < t < 0.88, 0.00 \text{ for otherwise} \quad (13)$$

The road input is described by Eq.(13) is shown in Fig.10.

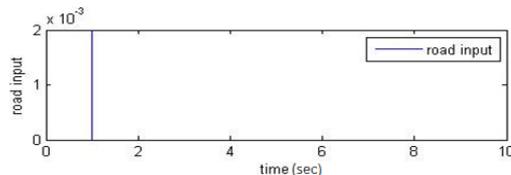
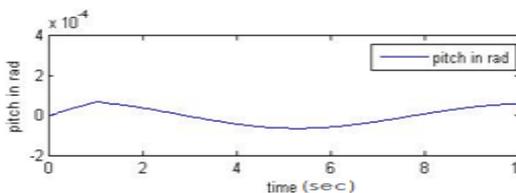


Fig.10 Step Input



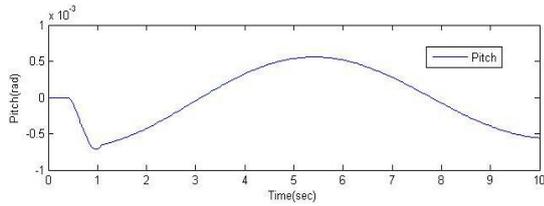


Fig.11 Pitch of Sprung Mass for Linear and Non-Linear Half Car Model

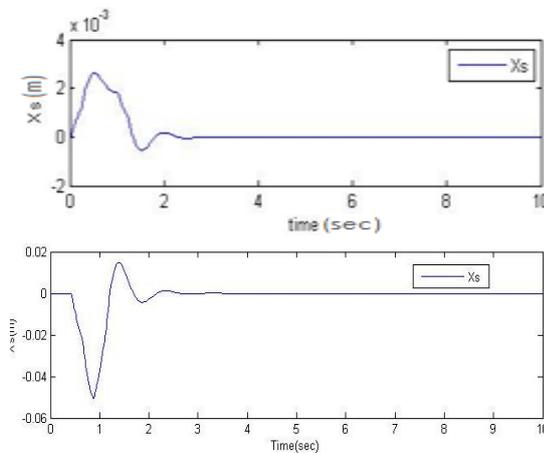


Fig.12 Sprung Mass Displacement (X_s) for Linear and Non-Linear Half Car Model

From the results obtained it is observed that the Pitch of Sprung Mass for Step Input is larger than Bumpy Input and Sprung Mass Displacement for Step input is less than Bumpy and Rectangular input.

Conclusion

From the results obtained by simulation, it is seen that it is important to consider nonlinear parameters. When graph of nonlinear passive suspension Model is compared with linear suspension Model for different road profiles, it is seen that behavior of nonlinear passive suspension Model tends to move towards the actual behavior of the Model. From the analysis of results, it is observed that the nonlinearities in the tire spring and tire damper does not affect much on ride comfort of vehicle, because of this, nonlinear effect included in those parameters can be neglected as it does not make much effect on sprung mass acceleration as compare to linear model of these parameters. But the nonlinearities in the suspension spring and suspension damper make considerable difference when compared with linear; hence nonlinearities in those must be considered while analyzing the vehicle dynamic model. RideComfort can be obtained by choosing proper damper.

Acknowledgements

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