

An Agricultural Economic Problem using Intuitionistic Fuzzy

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Abstract

The Concept of intuitionistic fuzzy set is the generalization of the concept of fuzzy sets is well suited to dealing with vagueness. In this paper, multi-attribute decision making using intuitionistic fuzzy set is investigated, in which multiple criteria are explicitly. In this paper, a new fuzzy TOPSIS decision making model using entropy weight for dealing with multiple criteria decision making (MCDM) problems under intuitionistic fuzzy environment is proposed. This model allows measuring the degree of satisfiability and the degree of non – satisfiability, respectively, of each alternative evaluated across a set of criteria. To obtain the weighted fuzzy decision matrix, the concept of Shannon's entropy is employed to calculate the criteria weights. Feasibility and effectiveness of the proposed method are illustrated using numerical example.

Keywords: Intuitionistic Fuzzy Set, Topsis

1. Introduction

The theory of fuzzy sets proposed by Zadeh has achieved a great success in various fields [7]. The concept of an intuitionistic fuzzy set (IFS) introduced by Atanassov has been found to be highly useful to deal with vagueness / imprecision. IFS theory has been extensively applied to areas like Artificial Intelligence, networking, Soft decision making, Programming logic, operational research etc. On the role of IFS has been used in decision making problems[6]. In some real – life situations, decision maker may not be able to accurately express their view for the problems as a precise or the decision maker are unable to discriminate explicitly the degree to which one alternative are better than others in such cases, the decision maker may provide their preferences for alternatives to a certain degree, but it is very suitable to express the decision maker preference values with the use of fuzzy / intuitionistic fuzzy values rather than exact numerical values or linguistic variables[10].

2. Preliminaries

2.1 Definition [8]

Let "X" be the universal set. A fuzzy set A in X represented by $A = \{(x, \mu_A(x)) / x \in X\}$, where the function $\mu_A(x): X \rightarrow [0,1]$ is the membership degree of element x in the fuzzy set X.

2.2 Definition [9]

An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of the membership and the degree of non-membership of the element $x \in X$ in A, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ holds.

For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A. Obviously, for every $x \in X$, $0 \leq \pi_A(x) \leq 1$.

Noted that A is a crisp set if and only if for $\forall x \in X$, either $\mu_A(x) = 0, \nu_A(x) = 1$ or $\mu_A(x) = 1, \nu_A(x) = 0$.

Note: Throughout this paper, μ represents membership values and ν represents non membership values.

2.3 Definition [8]

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set. Each numerical value in the domain is assigned a specific "grade of membership".

2.4 Definition [9]

An Intuitionistic Fuzzy Number (IFN) \tilde{A}^I is

- a) an intuitionistic fuzzy subset of the real line,
- b) convex for the membership function $\mu_{\tilde{A}'}(x)$, that $\mu_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}'}(x_1), \mu_{\tilde{A}'}(x_2))$, for every $x_1, x_2 \in R, \lambda \in [0,1]$.
- c) concave for the membership function $v_{\tilde{A}'}(x)$, that is, $v_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \leq \max(v_{\tilde{A}'}(x_1), v_{\tilde{A}'}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.
- d) normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}'}(x_0) = 1, v_{\tilde{A}'}(x_0) = 0$.

2.5 Definition [8]

A fuzzy number $\tilde{A} = (a, b, c, d)$ is called trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b; \\ 1 & b \leq x \leq c; \\ \frac{x-d}{c-d} & c \leq x \leq d; \\ 0 & \text{otherwise} \end{cases} \text{ where } a, b, c, d \in R.$$

2.6 Definition [9]

A Trapezoidal Intuitionistic Fuzzy Number (TIFN) \tilde{A}^I is an IFS in R with membership function and non-membership function as follows:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-(a_1-\alpha)}{\alpha} & \text{for } x \in [a_1-\alpha, a_1] \\ 1 & \text{for } x \in [a_1, a_2] \\ \frac{a_2+\beta-x}{\beta} & \text{for } x \in [a_2, a_2+\beta] \\ 0 & \text{otherwise} \end{cases}$$

$$v_{\tilde{A}^I}(x) = \begin{cases} \frac{a_1-x}{\alpha'} & \text{for } x \in [a_1-\alpha', a_1] \\ 0 & \text{for } x \in [a_1, a_2] \\ \frac{x-a_2}{\beta'} & \text{for } x \in [a_2, a_2+\beta'] \\ 1 & \text{otherwise} \end{cases}$$

2.7 Definition [4]

For every $A \in \text{IFSs}(X)$, the IFS λA for any positive real number λ is defined as follows:

$$\lambda A = \left\{ \left\langle x, 1 - (1 - \mu_A(x))^\lambda, (v_A(x))^\lambda \right\rangle / x \in X \right\}.$$

3. Proposed Method

Step:1

Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives. Each alternative is assessed on n criteria and the set of all criteria is denoted $C = \{C_1, C_2, \dots, C_n\}$. Let $W = \{w_1, w_2, \dots, w_n\}$ be the weighting vector of criteria, where $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$.

The characteristics of the alternatives A_i are represented by intuitionistic fuzzy set as follows:

$A_i = \{ \langle C_j, \mu_{A_i}(C_j), v_{A_i}(C_j) \rangle / C_j \in C \}, i=1, 2, \dots, m, j=1, 2, \dots, n$ where $\mu_{A_i}(C_j)$ and $v_{A_i}(C_j)$ indicate the degree of the membership and nonmembership of the alternatives A_i satisfies and does not satisfy the criterion C_j , $\mu_{A_i}(C_j) \in [0,1], v_{A_i}(C_j) \in [0,1]$ and $\mu_{A_i}(C_j) + v_{A_i}(C_j) \in [0,1]$. The intuitionistic index $\pi_{A_i}(C_j) = 1 - \mu_{A_i}(C_j) - v_{A_i}(C_j)$ is such that the larger $\pi_{A_i}(C_j)$ the higher a hesitation margin of the decision matrix about the alternative A_i with respect to the criterion C_j .

Step:2

In 1948, Shannon proposed the entropy function, for a measure of uncertainty in a discrete distribution based on the Boltzmann entropy of classical statistical mechanics, where the probabilities of random variable from a probability mass function. Then De Luca and Termini defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function. Later, Szmidt and Kacprzyk extended De Luca and Termini entropy measure for IFSs (X), denoted as $E_{LT}^{IFS}(C_j)$ as,

$$E_{LT}^{IFS}(C_j) = - \frac{1}{m \ln 2} \sum_{i=1}^m \left[\mu_{ij}(C_j) \ln \mu_{ij}(C_j) + v_{ij}(C_j) \ln v_{ij}(C_j) - (1 - \pi_{ij}(C_j)) \ln (1 - \pi_{ij}(C_j)) - \pi_{ij}(C_j) \ln 2 \right]$$

where $j = 1, 2, \dots, n$ and $1/(m \ln 2)$ is a constant with $0 \leq E_{LT}^{IFS}(C_j) \leq 1$. Then, the degree of divergence (d_j) provided as

$$d_j = 1 - E_{LT}^{IFS}(C_j), \quad j = 1, 2, \dots, n.$$

The value of d_j represents the inherent contrast intensity of criterion C_j , then the entropy weight is,

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j}$$

Step:3

The weighted intuitionistic fuzzy decision matrix \tilde{Z} can be obtained as,

$$\tilde{Z} = W \otimes [\tilde{x}_{ij}]_{m \times n} = [\hat{x}_{ij}]_{m \times n}$$

where $W = (w_1, w_2, \dots, w_j, \dots, w_n)$;

$$\hat{x}_{ij} = \langle \hat{\mu}_{ij}, \hat{\nu}_{ij} \rangle = \langle 1 - (1 - \mu_{ij})^{w_j}, \nu_{ij}^{w_j} \rangle, w_j > 0.$$

Step:4

To find the intuitionistic fuzzy positive-ideal solution and intuitionistic fuzzy negative-ideal solution can be defined as,

$$A^+ = \left\{ \left\langle C_j, \left(\max_i \hat{\mu}_{ij}(C_j) / j \in G, \left(\min_i \hat{\mu}_{ij}(C_j) / j \in B \right) \right), \left(\min_i \hat{\nu}_{ij}(C_j) / j \in G, \left(\max_i \hat{\nu}_{ij}(C_j) / j \in B \right) \right) \right\rangle / i \in m \right\}$$

$$A^- = \left\{ \left\langle C_j, \left(\min_i \hat{\mu}_{ij}(C_j) / j \in G, \left(\max_i \hat{\mu}_{ij}(C_j) / j \in B \right) \right), \left(\max_i \hat{\nu}_{ij}(C_j) / j \in G, \left(\min_i \hat{\nu}_{ij}(C_j) / j \in B \right) \right) \right\rangle / i \in m \right\}$$

where G be the collection of benefit criteria and B be a collection of cost criteria.

Step:5

The distance measures of each alternative A_i from IFPIS and IRNIS using the measure of intuitionistic Euclidean distance refer to Szmidt and Kacprzyk determined as,

$$d_{IFS}(A_i, A^+) = \sqrt{\sum_{j=1}^n \left[(\mu_{A_i}(C_j) - \mu_{A^+}(C_j))^2 + (\nu_{A_i}(C_j) - \nu_{A^+}(C_j))^2 + (\pi_{A_i}(C_j) - \pi_{A^+}(C_j))^2 \right]}$$

$$d_{IFS}(A_i, A^-) = \sqrt{\sum_{j=1}^n \left[(\mu_{A_i}(C_j) - \mu_{A^-}(C_j))^2 + (\nu_{A_i}(C_j) - \nu_{A^-}(C_j))^2 + (\pi_{A_i}(C_j) - \pi_{A^-}(C_j))^2 \right]}$$

Step:6

The relative closeness coefficient of each alternatives with the intuitionistic fuzzy ideal solution is determined as,

$$CC_i = \frac{d_{IFS}(A_i, A^-)}{d_{IFA}(A_i, A^+) + d_{IFS}(A_i, A^-)}$$

where $0 \leq CC_i \leq 1, i = 1, 2, \dots, m.$

Step:7

The large value of alternative is the one with the highest CC_i . Therefore, the ranking order of all the alternatives can be determined by descending order.

4. Numerical Example

An agricultural cost of cultivation problem is considered where five different types of crops such as Paddy,

Cholam, Maize, Blackgram and groundnut are considered as alternatives as A_1, A_2, A_3, A_4, A_5 and analysed with the criteria Ploughing and Land cost, Fertilizer and Manures, Irrigation charges, Cost of labour and Income from the crop as C_1, C_2, C_3, C_4 and C_5 respectively.

Table:1 Basic Data

	Ploughing and Land cost	Fertilizers and Manures	Irrigation charges	Cost of Production	Income from the Crop
Paddy	9093	7934	332	1424	182000
Cholam	2932	2404	0	1138	31500
Maize	7345	11032	332	1287	108000
Blackgram	2513	2093	0	4314	32600
Groundnut	3523	7344	1838	2962	115000

Step:1

To find the intuitionistic fuzzy decision matrix here consider the Trapezoidal Intuitionistic Fuzzy Number.

Rating	Range	Linguistic variables	Fuzzy Rating
1	0 to 2,500	Low	(0.0,0.0,0.1,0.2)
3	2,501 to 10,000	Fair	(0.2,0.3,0.4,0.5)
5	10,001 to 50,000	Medium	(0.4,0.5,0.6,0.7)
7	50,001 to 1,10,000	High	(0.5,0.6,0.7,0.8)
9	1,10,001 to 2,00,000	Very High	(0.6,0.7,0.8,0.9)

Table:2 Intuitionistic Fuzzy Decision Matrix

Criteria	C_1	C_2	C_3	C_4	C_5
Alternatives					
A_1	<0.75, 0.04>	<0.6, 0.14>	<0.05, 0.32>	<0.19, 0.27>	<0.97, 0.02>
A_2	<0.33, 0.24>	<0.27, 0.24>	<0.01, 0.33>	<0.11, 0.30>	<0.53, 0.35>
A_3	<0.58, 0.16>	<0.34, 0.50>	<0.05, 0.32>	<0.15, 0.28>	<0.88, 0.09>
A_4	<0.31, 0.22>	<0.25, 0.25>	<0.01, 0.33>	<0.42, 0.27>	<0.58, 0.31>
A_5	<0.35, 0.25>	<0.58, 0.16>	<0.23, 0.26>	<0.34, 0.24>	<0.95, 0.03>

Step:2

$$E_{LT}^{IFS}(C_1) = -0.2885[-0.3039-0.6860-0.5666-0.6855-0.6848] = 0.8444$$

Table: 3 Entropy Weight Calculations

	C ₁	C ₂	C ₃	C ₄	C ₅
$E_{IT}^{IFS}(C_j)$	0.8444	0.9143	0.8262	0.9717	0.548
d_j	0.1556	0.0857	0.1738	0.0283	0.452
W	0.1738	0.0957	0.1941	0.0316	0.5048

$$\sum_{j=1}^n d_j = 0.8954$$

Step:3

Table:4 Weighted Intuitionistic Fuzzy Decision Matix

Criteria	C ₁	C ₂	C ₃	C ₄	C ₅
Alternatives					
A ₁	<0.2141, 0.5715>	<0.0840, 0.8285>	<0.0099, 0.8016>	<0.0066, 0.9595>	<0.8297, 0.1388>
A ₂	<0.0672, 0.7803>	<0.0297, 0.8723>	<0.0019, 0.8064>	<0.0037, 0.9627>	<0.3169, 0.5886>
A ₃	<0.1400, 0.7272>	<0.0390, 0.9358>	<0.0099, 0.8016>	<0.0051, 0.9606>	<0.6571, 0.2966>
A ₄	<0.0625, 0.7686>	<0.0272, 0.8758>	<0.0019, 0.8064>	<0.0171, 0.9595>	<0.3546, 0.5537>
A ₅	<0.0721, 0.7859>	<0.0797, 0.8391>	<0.0495, 0.7699>	<0.0130, 0.9559>	<0.7796, 0.1703>

Step:4

$$A^+ = \{(0.2141,0.5715), (0.0840,0.8285), (0.0495,0.7699), (0.0171,0.9595), (0.8297,0.1388)\}$$

$$A^- = \{(0.0625,0.7686), (0.0272,0.8758), (0.0019,0.8064), (0.0037,0.9627), (0.3169,0.5886)\}$$

Step:5

Table:5 Relative Closeness Coefficient and Ranking

	$d_{IFS}(A_i, A^+)$	$d_{IFS}(A_i, A^-)$	CC _i	Rank
A ₁	0.0534	0.734	0.9322	1
A ₂	0.7396	0.0079	0.0106	5
A ₃	0.3337	0.4705	0.5851	3
A ₄	0.689	0.0542	0.0729	4
A ₅	0.2775	0.6327	0.6951	2

Conclusions

In this paper a view on the role of intuitionistic fuzzy set in decision making problem is considered. As this decision making problem face many problems with vague situations to satisfy the need of decision making problem with imprecision and uncertainty many researchers have been concentrated on IFS theory. The optimal value is obtained using entropy weight in intuitionistic fuzzy environment. Using the proposed method the order of ranking is $A_1 > A_5 > A_3 > A_4 > A_2$. The highest value of A_5 (Paddy) is the best choice represent the criteria's are ploughing and land cost, fertilizers and manures, irrigation charges, cost of production and income from the crop.

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