An Agricultural Economic Problem using Intuitionistic Fuzzy

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Accepted 01 Sept 2015, Available online 06 Sept 2015, Vol.3 (Sept/Oct 2015 issue)

Abstract

The concept of intuitionistic fuzzy set is the generalization of the concept of fuzzy sets is well suited to dealing with vaguesness. In this paper, multi-attribute decision making using intuitionistic fuzzy set is investigated, in which multiple criteria are explicitly. In this paper, a new fuzzy TOPSIS decision making model using entropy weight for dealing with multiple criteria decision making (MCDM) problems under intuitionistic fuzzy environment is proposed. This model allows measuring the degree of satisfiability and the degree of non-satisfiability, respectively, of each alternative evaluated across a set of criteria. To obtain the weighted fuzzy decision matrix, the concept of Shannon’s entropy is employed to calculate the criteria weights. Feasibility and effectiveness of the proposed method are illustrated using numerical example.

Keywords: Intuitionistic Fuzzy Set, TOPSIS

1. Introduction

The theory of fuzzy sets proposed by Zadeh has achieved a great success in various fields [7]. The concept of an intuitionistic fuzzy set (IFS) introduced by Atanassov has been found to be highly useful to deal with vagueness/imprecision. IFS theory has been extensively applied to areas like Artificial Intelligence, networking, Soft decision making, Programming logic, operational research etc. On the role of IFS has been used in decision making problems[6]. In some real-life situations, decision makers may not be able to accurately express their view for the problems as a precise or the decision makers are unable to discriminate explicitly the degree to which one alternative are better than others in such cases, the decision maker may provide their preferences for alternatives to a certain degree, but it is very suitable to express the decision maker preference values with the use of fuzzy/intuitionistic fuzzy values rather than exact numerical values or linguistic variables[10].

2. Preliminaries

2.1 Definition [8]

Let "X" be the universal set. A fuzzy set A in X represented by $A = \{ (x, \mu_A(x)) | x \in X \}$, where the function $\mu_A(x): X \rightarrow [0,1]$ is the membership degree of element $x$ in the fuzzy set $X$.

2.2 Definition [9]

An Intuitionistic Fuzzy Set (IFS) $A$ in $X$ is defined as an object of the form $A = \{ (x, \mu_A(x), v_A(x)) | x \in X \}$ where the functions $\mu_A: X \rightarrow [0,1]$ and $v_A: X \rightarrow [0,1]$ define the degree of the membership and the degree of non-membership of the element $x \in X$ in $A$, $0 \leq \mu_A(x) + v_A(x) \leq 1$ holds.

For every common fuzzy subset $A$ on $X$, Intuitionistic Fuzzy Index of $x$ in $A$ is defined as $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element $x$ in $A$. Obviously, for every $x \in X$, $0 \leq \pi_A(x) \leq 1$.

Noted that $A$ is a crisp set if and only if for $\forall x \in X$, either $\mu_A(x) = 0, v_A(x) = 1$ or $\mu_A(x) = 1, v_A(x) = 0$. 

Note: Throughout this paper, $\mu$ represents membership values and $v$ represents non membership values.

2.3 Definition [8]

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with “ordinary” (single valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set. Each numerical value in the domain is assigned a specific “grade of membership”.

2.4 Definition [9]

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}$ is

\[ \tilde{A} = (\mu_A, v_A) \]
a) an intuitionistic fuzzy subset of the real line, 

b) convex for the membership function \( \mu_{\tilde{x}}(x) \), that

\[
\mu_{\tilde{x}}(x_i + (1-\lambda)x_j) \geq \min\{\mu_{\tilde{x}}(x_i), \mu_{\tilde{x}}(x_j)\}, \text{ for every } x_i, x_j \in R, \lambda \in [0,1].
\]

c) concave for the membership function \( v_{\tilde{x}}(x) \), that is,

\[
v_{\tilde{x}}(x_i + (1-\lambda)x_j) \leq \max\{v_{\tilde{x}}(x_i), v_{\tilde{x}}(x_j)\}, \text{ for every } x_i, x_j \in R, \lambda \in [0,1].
\]

d) normal, that is, there is some \( x_0 \in R \) such that

\[
\mu_{\tilde{x}}(x_0) = 1, \quad v_{\tilde{x}}(x_0) = 0.
\]

2.5 Definition [8]

A fuzzy number \( \tilde{A} = (a, b, c, d) \) is called trapezoidal fuzzy number if its membership function is given by,

\[
\mu_{\tilde{x}}(x)=
\begin{cases}
\frac{x-a}{b-a} & \text{for } a \leq x \leq b; \\
1 & \text{for } b \leq x \leq c; \\
\frac{c-x}{d-c} & \text{for } c \leq x \leq d; \\
0 & \text{otherwise}
\end{cases}
\]

where \( a, b, c, d \in R \).

2.6 Definition [9]

A Trapezoidal Intuitionistic Fuzzy Number (TIFN) \( \tilde{A}^T \) is an IFS in \( R \) with membership function and non-membership function as follows:

\[
\mu_{\tilde{x}}(x)=
\begin{cases}
\frac{x-(a_1-a)}{a} & \text{for } x \in [a_1-a, a_1] \\
1 & \text{for } x \in [a_1, a_2] \\
\frac{a_2-x}{\beta} & \text{for } x \in [a_1, a_2 + \beta] \\
0 & \text{otherwise}
\end{cases}
\]

\[
v_{\tilde{x}}(x)=
\begin{cases}
\frac{a_1-x}{\alpha} & \text{for } x \in [a_1, a_1^\prime, a_1] \\
0 & \text{for } x \in [a_1^\prime, a_2] \\
\frac{x-a_2}{\beta} & \text{for } x \in [a_2, a_2 + \beta] \\
1 & \text{otherwise}
\end{cases}
\]

2.7 Definition [4]

For every \( A \in \text{IFSs}(X) \), the IFS \( \lambda A \) for any positive real number \( \lambda \) is defined as follows:

\[
\lambda A = \left\{ (x, 1-(1-\mu_{\lambda}(x))^{1/\lambda}, (v_{\lambda}(x))^{1/\lambda}) / x \in X \right\}
\]

3. Proposed Method

Step:1

Let \( A = \{A_1, A_2, \ldots, A_m\} \) be a set of alternatives. Each alternative is assessed on \( n \) criteria and the set of all criteria is denoted \( C = \{C_1, C_2, \ldots, C_n\} \). Let \( w = \{w_1, w_2, \ldots, w_n\} \) be the weighting vector of criteria, where \( w_j \geq 0 \) and \( \sum_{j=1}^{n} w_j = 1 \).

The characteristics of the alternatives \( A \) are represented by intuitionistic fuzzy set as follows:

\[
A_i = (\{C_j, \mu_{A_i}(C_j), v_{A_i}(C_j) / C_j \in C\}) \quad i=1,2,\ldots,m. \quad j=1,2,\ldots,n.
\]

where \( \mu_{A_i}(C_j) \) and \( v_{A_i}(C_j) \) indicate the degree of the membership and nonmembership of the alternatives \( A \) satisfies and does not satisfy the criterion \( C_j \), \( \mu_{A_i}(C_j) \in [0,1], v_{A_i}(C_j) \in [0,1] \) and \( \mu_{A_i}(C_j) + v_{A_i}(C_j) \in [0,1] \).

The intuitionistic index \( \pi_{\lambda}(C_j) = 1 - \mu_{A_i}(C_j) - v_{A_i}(C_j) \) is such that the larger \( \pi_{\lambda}(C_j) \) the higher a hesitation margin of the decision matrix about the alternative \( A \) with respect to the criterion \( C_j \).

Step:2

In 1948, Shannon proposed the entropy function, for a measure of uncertainty in a discrete distribution based on the Boltzmann entropy of classical statistical mechanics, where the probabilities of random variable from a probability mass function. Then De Luca and Termini defined a non-probabilistic entropy formula of a fuzzy set based on Shannon’s function. Later, Szmidy and Kacprzyk extended De Luca and Termini entropy measure for IFSs \( (X) \), denoted as \( E^{BS}_{LT}(C_j) \) as,

\[
E^{BS}_{LT}(C_j) = -\frac{1}{m \ln 2} \sum_{j=1}^{n} \left[ \mu_{A_i}(C_j) \ln \mu_{A_i}(C_j) + v_{A_i}(C_j) \ln v_{A_i}(C_j) \right]
\]

where \( j = 1, 2, \ldots, n \) and \( 1/(m \ln 2) \) is a constant with \( 0 \leq E^{BS}_{LT}(C_j) \leq 1 \). Then, the degree of divergence \( (d_j) \) provided as

\[
d_j = 1 - E^{BS}_{LT}(C_j), \quad j=1,2,\ldots,n.
\]

The value of \( d_j \) represents the inherent contrast intensity of criterion \( C_j \), then the entropy weight is,

\[
w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}
\]
Step:3

The weighted intuitionistic fuzzy decision matrix $\tilde{Z}$ can be obtained as,

$$\tilde{Z} = W \otimes [\hat{x}_{ij}] = [\hat{x}_{ij}]$$

where $W = (w_1, w_2, \ldots, w_j, \ldots, w_n)$;

$$\hat{x}_{ij} = (\hat{\mu}_{ij}, \hat{\nu}_{ij}) = \{1 - (1 - \mu_{ij})^+, \nu_{ij}^w\}, \quad w_j > 0.$$

Step:4

To find the intuitionistic fuzzy positive-ideal solution and intuitionistic fuzzy negative-ideal solution can be defined as,

$$A^+ = \left\{ \sum_{i=1}^{m} \left( \max_{j \in G} \hat{\mu}_{ij}(C_j) \right), \sum_{i=1}^{m} \left( \min_{j \in G} \hat{\nu}_{ij}(C_j) \right) \right\}$$

$$A^- = \left\{ \sum_{i=1}^{m} \left( \min_{j \in G} \hat{\mu}_{ij}(C_j) \right), \sum_{i=1}^{m} \left( \max_{j \in G} \hat{\nu}_{ij}(C_j) \right) \right\}$$

where $G$ be the collection of benefit criteria and $B$ be a collection of cost criteria.

Step:5

The distance measures of each alternative $A_j$ from IFPIS and IRNIS using the measure of intuitionistic Euclidean distance refer to Szmidt and Kacprzyk determined as,

$$d_{\mu}(A_j, A^+) = \sqrt{\sum_{i=1}^{m} \left( \mu_{ij}(C_j) - \mu^+(C_j) \right)^2 + (\nu_{ij}(C_j) - \nu^-(C_j))^2}$$

$$d_{\nu}(A_j, A^-) = \sqrt{\sum_{i=1}^{m} \left( \mu_{ij}(C_j) - \mu^+(C_j) \right)^2 + (\nu_{ij}(C_j) - \nu^-(C_j))^2}$$

Step:6

The relative closeness coefficient of each alternatives with the intuitionistic fuzzy ideal solution is determined as,

$$CC_i = \frac{d_{\mu}(A_i, A^+)}{d_{\mu}(A_i, A^+) + d_{\nu}(A_i, A^-)}$$

where $0 \leq CC_i \leq 1, \quad i = 1, 2, \ldots, m.$

Step:7

The large value of alternative is the one with the highest $CC_i$. Therefore, the ranking order of all the alternatives can be determined by descending order.

4. Numerical Example

An agricultural cost of cultivation problem is considered where five different types of crops such as Paddy, Cholam, Maize, Blackgram and groundnut are considered as alternatives $A_1, A_2, A_3, A_4, A_5$ and analysed with the criteria Ploughing and Land cost, Fertilizer and Manures, Irrigation charges, Cost of labour and Income from the crop as $C_1, C_2, C_3, C_4$ and $C_5$ respectively.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Ploughing and Land cost</th>
<th>Fertilizers and Manures</th>
<th>Irrigation charges</th>
<th>Cost of Production</th>
<th>Income from the Crop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paddy</td>
<td>9093</td>
<td>7934</td>
<td>332</td>
<td>1424</td>
<td>182000</td>
</tr>
<tr>
<td>Cholam</td>
<td>2932</td>
<td>2404</td>
<td>0</td>
<td>1138</td>
<td>31500</td>
</tr>
<tr>
<td>Maize</td>
<td>7345</td>
<td>11032</td>
<td>332</td>
<td>1287</td>
<td>108000</td>
</tr>
<tr>
<td>Blackgram</td>
<td>2513</td>
<td>2093</td>
<td>0</td>
<td>4314</td>
<td>32600</td>
</tr>
<tr>
<td>Groundnut</td>
<td>3523</td>
<td>7344</td>
<td>1838</td>
<td>2962</td>
<td>115000</td>
</tr>
</tbody>
</table>

Step:1

To find the intuitionistic fuzzy decision matrix here consider the Trapezoidal Intuitionistic Fuzzy Number.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Range</th>
<th>Linguistic variables</th>
<th>Fuzzy Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 to 2,500</td>
<td>Low</td>
<td>(0.0, 0.0, 0.1, 0.2)</td>
</tr>
<tr>
<td>3</td>
<td>2,501 to 10,000</td>
<td>Fair</td>
<td>(0.2, 0.3, 0.4, 0.5)</td>
</tr>
<tr>
<td>5</td>
<td>10,001 to 50,000</td>
<td>Medium</td>
<td>(0.4, 0.5, 0.6, 0.7)</td>
</tr>
<tr>
<td>7</td>
<td>50,001 to 110,000</td>
<td>High</td>
<td>(0.5, 0.6, 0.7, 0.8)</td>
</tr>
<tr>
<td>9</td>
<td>1,10,001 to 2,00,000</td>
<td>Very high</td>
<td>(0.6, 0.7, 0.8, 0.9)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>&lt;0.75</td>
<td>&lt;0.6</td>
<td>&lt;0.05</td>
<td>&lt;0.19</td>
<td>&lt;0.97</td>
</tr>
<tr>
<td>$A_2$</td>
<td>&lt;0.33</td>
<td>&lt;0.27</td>
<td>&lt;0.01</td>
<td>&lt;0.11</td>
<td>&lt;0.53</td>
</tr>
<tr>
<td>$A_3$</td>
<td>&lt;0.58</td>
<td>&lt;0.34</td>
<td>&lt;0.05</td>
<td>&lt;0.15</td>
<td>&lt;0.88</td>
</tr>
<tr>
<td>$A_4$</td>
<td>&lt;0.31</td>
<td>&lt;0.25</td>
<td>&lt;0.01</td>
<td>&lt;0.42</td>
<td>&lt;0.58</td>
</tr>
<tr>
<td>$A_5$</td>
<td>&lt;0.35</td>
<td>&lt;0.28</td>
<td>&lt;0.23</td>
<td>&lt;0.34</td>
<td>&lt;0.95</td>
</tr>
</tbody>
</table>

Step:2

$$E_{\mu_j}^C(C_i) = -0.2885[-0.3039-0.6860-0.5666-0.6855-0.6848] = 0.8444$$

The highest value of $A_3$ is the best choice represent the criteria’s are ploughing and land cost, fertilizers and manures, irrigation charges, cost of production and income from the crop.

### Table 3: Entropy Weight Calculations

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{TT}^{(C_j)}$</td>
<td>0.8444</td>
<td>0.9143</td>
<td>0.8262</td>
<td>0.9717</td>
<td>0.548</td>
</tr>
<tr>
<td>$d_j$</td>
<td>0.1556</td>
<td>0.0857</td>
<td>0.1738</td>
<td>0.0283</td>
<td>0.452</td>
</tr>
<tr>
<td>$W$</td>
<td>0.1738</td>
<td>0.0957</td>
<td>0.1941</td>
<td>0.0316</td>
<td>0.5048</td>
</tr>
</tbody>
</table>

$$\sum_{j=1}^{n} d_j = 0.8954$$

### Step 3

Table 4: Weighted Intuitionistic Fuzzy Decision Matrix

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>&lt;0.2141, 0.5715&gt;</td>
<td>&lt;0.0840, 0.8285&gt;</td>
<td>&lt;0.0099, 0.8016&gt;</td>
<td>&lt;0.0066, 0.9595&gt;</td>
<td>&lt;0.8297, 0.1388&gt;</td>
</tr>
<tr>
<td>$A_2$</td>
<td>&lt;0.0672, 0.7803&gt;</td>
<td>&lt;0.0297, 0.8723&gt;</td>
<td>&lt;0.0019, 0.8064&gt;</td>
<td>&lt;0.0037, 0.9627&gt;</td>
<td>&lt;0.3169, 0.5886&gt;</td>
</tr>
<tr>
<td>$A_3$</td>
<td>&lt;0.1400, 0.7272&gt;</td>
<td>&lt;0.0390, 0.9358&gt;</td>
<td>&lt;0.0099, 0.8016&gt;</td>
<td>&lt;0.0051, 0.9606&gt;</td>
<td>&lt;0.6571, 0.2966&gt;</td>
</tr>
<tr>
<td>$A_4$</td>
<td>&lt;0.0625, 0.7686&gt;</td>
<td>&lt;0.0272, 0.8758&gt;</td>
<td>&lt;0.0019, 0.8064&gt;</td>
<td>&lt;0.0171, 0.9595&gt;</td>
<td>&lt;0.3546, 0.5537&gt;</td>
</tr>
<tr>
<td>$A_5$</td>
<td>&lt;0.0721, 0.7859&gt;</td>
<td>&lt;0.0797, 0.8391&gt;</td>
<td>&lt;0.0495, 0.7699&gt;</td>
<td>&lt;0.0130, 0.9559&gt;</td>
<td>&lt;0.7796, 0.1703&gt;</td>
</tr>
</tbody>
</table>

### Step 4

$A^+ = \{(0.2141,0.5715), (0.0840,0.8285), (0.0495,0.7699), (0.0171,0.9595), (0.8297,0.1388)\}$

$A^- = \{(0.0625,0.7686), (0.0272,0.8758), (0.0019,0.8064), (0.0037,0.9627), (0.3169,0.5886)\}$

### Step 5

Table 5: Relative Closeness Coefficient and Ranking

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$d_{IFS}(A^+) - d_{IFS}(A^-)$</th>
<th>$CC_i$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.0534</td>
<td>0.734</td>
<td>0.9322</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.7396</td>
<td>0.0079</td>
<td>0.0106</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.3337</td>
<td>0.4705</td>
<td>0.5851</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.689</td>
<td>0.0542</td>
<td>0.0729</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.2775</td>
<td>0.6327</td>
<td>0.6951</td>
</tr>
</tbody>
</table>

### Conclusions

In this paper a view on the role of intuitionistic fuzzy set in decision making problem is considered. As this decision making problem face many problems with vague situations to satisfy the need of decision making problem with imprecision and uncertainty many researchers have been concentrated on IFS theory. The optimal value is obtained using entropy weight in intuitionistic fuzzy environment. Using the proposed method the order of ranking is $A_1 > A_3 > A_4 > A_5$. The highest value of $A_3$ (Paddy) is the best choice represent the criteria’s are ploughing and land cost, fertilizers and manures, irrigation charges, cost of production and income from the crop.

### References

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