International Journal of Multidisciplinary and Current Research

Research Article

ISSN: 2321-3124
Available at: http://ijmcr.com

Creation of Optimal Path using Min-sum Algebra Technique

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Accepted 18 Oct 2015, Available online 25 Oct 2015, **Vol.3 (Sept/Oct 2015 issue)**

Abstract

The main focus of this paper is to examine the impact of Min-sum Algebra Method on real road network to optimize/minimize the real distances .Two major key factors are pursued in this paper. Firstly, Min-sum Algebra Method is applied to optimize/minimize the distances. Secondly, the shortest routes or paths composed of any particular network by using forward and backward tracking are found. The comparison of forward and backward tracking is also presented in this paper. Min-sum Algebra method enables to provide shortest or minimum distances. We used Min-sum algebra and Structural matrix to generate Optimal Matrix (OM) and Initial Matrix (IM) respectively. Using the concept of forward and backward tracking on IM and OM shortest routes or paths are calculated.

Keywords: *Creation of Shortest Path Problems, Min-sum Algebra technique, Structure matrix, weighted Graph.*

1. Introduction

1.1 Graph theory is a most important branch of Applied and computational mathematics shows how networks can be encoded and how their properties are measured. In last decades, it has been developed by growing effects from studies of social and complex networks. For example a design of data structure can be in the form of tree which in turn operated vertices or nodes and edges. On the other hand network modeling topologies can be prepared by using graphical ideas contained by the same approach the foremost vital conception of graph coloring is used in arrangement and resource allocation. Also, In graph theory walks, path and circuits are used in incredible applications say concepts of database design, salesman traveling difficulty and resources of networking.

1.2 History: The history of graph theory was initially traced in 1735, when Leonhard Euler, the Swiss mathematician solved the Konigsberg bridge problem. This study gives the concept of Eulerian Graph. He considered the problem of Konigsberg Bridge and created a structure of problem to solve this Eulerian Graph [9].
In 1840, A.F Mobius and Kuratowski gave idea of complete graph and bipartite graph and also gave the proof of it.[1]

In 1845, the idea of tree, (a connected graph without cycles) was applied by Gustav Kirchhoff and he gives the theoretical concepts of currents calculations in electrical circuits or networks.

In 1852,The famous four color problem establish by Thomas Gutherie[1], after this in 1856,Thomas. P. Kirkman and William R.Hamilton considered cycles on polyhydra and developed the theory called Hamiltonian graph by learning journeys that go to definite sites precisely once [1].

In 1913, apuzzle problem was specified by H.Dudeney. Then Kenneth Appel and Wolfgang Haken solved the same problem throughout the century which was considered as the beginning of Graph Theory [1].

Caley considered specific systematic forms from insignificant calculus for tree reviews. In theoretical chemistry, it had numerous simplifications. This effect shows the innovation of enumerative graph theory nevertheless in 1878 Sylvester presented the term "Graph" wherever he historian associate between "Quantic invariants" and molecular diagrams and covariates of pure mathematics in Nursing analogy[1].

Ramsey operated on colorations in 1941 that result in the identification of additional branch of graph theory known as external graph theory. The four color drawback was resolved with the help of optimization computers by Heinrich 1969. The phenomena of straight line graph property gave growth to random graph theory [1].

1.3 Uses of Graph Theory: In various applications and in numerous areas graph theories ideas are commonly used to study and model. They embrace construction of bonds in chemistry, molecules study and also the study of atoms. In the same way, graph theory is used in social science as an example to live actor's status or to discover circulation mechanisms. In conservation efforts and ecology graph theory is also used wherever a node or vertex signifies regions, and also the edges [10].This evidence is very indispensable on observing pursuit the unfolding of malady or breeding patterns, organisms and to review the power of migration that have an effect on alternative species.

Graph theories ideas are widely used in research. As an example, the representative downside, in a weighted graph the shortest spanning tree, getting associate degree optimum match of men and jobs and finding the shortest path between two nodes or vertices in a graph. In transport modeling networks it is additionally employed, measures of hypothesis of games and networks. The network action is employed to unravel sizable amount of combinatorial issues. In Operational research the foremost common and effective applications are networks designing and planning of huge difficult comes. The well-known method CPM (Critical Path Method) and PERT (Project Evaluation and Review Technique) and Next, In engineering, social science and war science game theory is also applied to seek out optimum thanks to perform certain tasks in reasonable environments. To signify the approach of finite game alphabetic character is used. Here, positions represent the vertices or nodes and moves represent the edges [1].

2. Objective

The objective of this research is to optimize the distances on real road network. Min-sum algebra is applied to minimize and optimize the distances for both single-source and all-pairs shortest path problems.

3. Method

3.1 Min-sum Algebra

On Information Networks in New York, Author introduced the following 'min-sum algebra':[3]

Arithmetic:

For any arbitrary infinite or real numbers x and y,

$x + y \equiv \min(x, y)$

and $xy \equiv$ the algebraic sum of x and y.

In [3] the above mentioned arithmetic is evaluated to the matrix product. Naming the distance matrix associated with a given length of matrix S the 'dispersion'.
In [3] stated:

"It follows trivially that Sk for k ≥ 1 is a matrix giving the shortest paths from site to site in S given that k − 1 other sites may be traversed in the process. It also follows that for any S there exists an integer k such that Sk = Sk+1. Clearly, the dispersion of S (let us label it D(S)) will be the matrix Sk such that
Sk = Sk+1."

In simple words he applied min sum algebra to nxn initial matrix and by using geometric series performs it to n-1 times, n-1 matrix is optimal solution and all the entries in optimal matrix are the shortest path.

3.2 Construction of Initial Matrix (IM) and Optimal Matrix (OM)

Initial matrix is developed from the given network and its entries depend upon the weights of the network which are written on the edges of network. Optimal matrix is established from the initial matrix after performing various computational techniques .In this paper; we used Min-sum Algebra techniques to achieved OM. In Graph theory OM tells us the shortest distance between any two particular nodes.

To construct IM and OM we used structural matrix which is a square matrix (symmetric/asymmetry matrix) for directed and undirected graphs. The construction for undirected graph of a square matrix is defined as:

$$A = \begin{cases} \infty & \text{if edges are not connected} \\ 0 & \text{if } i = j \text{ (no loop at any node)} \\ \text{weights} & \text{otherwise} \end{cases}$$

It gives '∞' if edges are not connected and ifedges are connected, it defines the given weights and gives '0' on main diagonal element of matrix.
Min-Sum Algebra is applied on IM till OM is achieved. The procedure of Min-sum algebra is minimized using Geometric Progression i.e. A^2, A^4, A^8 ……If the network has ***n*** node then A^{n-1} is required for achieving OM.

4. Result and Discussion

4.1 Application of Min-sum Algebra

Min-sum Algebra is easy to implement on maps, traffic routes, airport destination and many others networks. This method can be applied on weighted graphs to find shortest path or shortest distance for any type of networks graph theory.

Consider a random network of ten vertices or nodes as shown in fig 1.

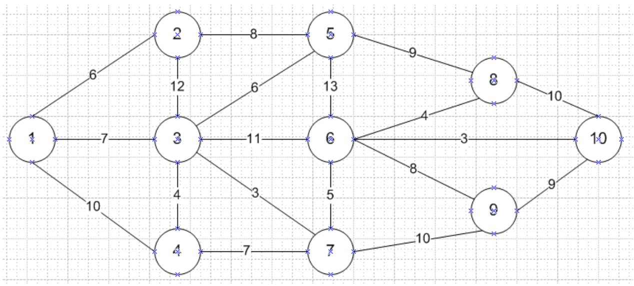

Figure 1 Random Network of ten vertices to construct Initial Matrix

According to section 3.2, Structural matrix of network in fig1 is shown below

$$A \times A = \begin{bmatrix} 0 & 6 & 7 & 10 & 13 & 18 & 10 & \infty & \infty & \infty \\ 6 & 0 & 12 & 16 & 8 & 21 & 15 & 17 & \infty & \infty \\ 7 & 12 & 0 & 4 & 6 & 8 & 3 & 12 & 13 & 11 \\ 10 & 16 & 4 & 0 & 10 & 12 & 7 & \infty & 17 & \infty \\ 13 & 8 & 6 & 10 & 0 & 13 & 9 & 9 & 21 & 16 \\ 18 & 21 & 8 & 12 & 13 & 0 & 5 & 4 & 8 & 3 \\ 10 & 13 & 3 & 7 & 9 & 5 & 0 & 9 & 10 & 8 \\ \infty & 17 & 12 & \infty & 9 & 4 & 9 & 0 & 12 & 7 \\ \infty & \infty & 13 & 17 & 21 & 8 & 10 & 12 & 0 & 9 \\ \infty & \infty & 11 & \infty & 16 & 3 & 8 & 7 & 9 & 0 \end{bmatrix}$$

Applying min sum Algebra as mentioned below

$A^2 \times A^2 =$

$$\begin{bmatrix} 0 & 6 & 7 & 10 & 13 & 18 & 10 & \infty & \infty & \infty \\ 6 & 0 & 12 & 16 & 8 & 21 & 15 & 17 & \infty & \infty \\ 7 & 12 & 0 & 4 & 6 & 8 & 3 & 15 & 13 & 14 \\ 10 & 16 & 4 & 0 & 10 & 12 & 7 & \infty & 17 & \infty \\ 13 & 8 & 6 & 10 & 0 & 13 & 9 & 9 & 21 & 16 \\ 18 & 21 & 8 & 12 & 13 & 0 & 5 & 4 & 8 & 3 \\ 10 & 15 & 3 & 7 & 9 & 5 & 0 & 9 & 10 & 8 \\ \infty & 17 & 12 & \infty & 9 & 4 & 9 & 0 & 12 & 7 \\ \infty & \infty & 13 & 17 & 21 & 8 & 10 & 12 & 0 & 9 \\ \infty & \infty & 11 & \infty & 16 & 3 & 8 & 7 & 9 & 0 \end{bmatrix}$$

$A^4 \times A^4 =$

$$\begin{bmatrix} 0 & 6 & 7 & 10 & 13 & 15 & 10 & 19 & 20 & 18 \\ 6 & 0 & 12 & 16 & 8 & 20 & 15 & 17 & 25 & 23 \\ 7 & 12 & 0 & 4 & 6 & 8 & 3 & 12 & 13 & 11 \\ 10 & 16 & 4 & 0 & 10 & 12 & 7 & 16 & 17 & 15 \\ 13 & 8 & 6 & 10 & 0 & 13 & 9 & 9 & 19 & 16 \\ 15 & 20 & 8 & 12 & 13 & 0 & 5 & 4 & 8 & 3 \\ 10 & 15 & 3 & 7 & 9 & 5 & 0 & 9 & 10 & 8 \\ 19 & 17 & 12 & 16 & 9 & 4 & 9 & 0 & 12 & 7 \\ 20 & 25 & 13 & 17 & 19 & 8 & 10 & 12 & 0 & 9 \\ 18 & 23 & 11 & 15 & 16 & 3 & 8 & 7 & 9 & 0 \end{bmatrix}$$

$.A^9 \times A^9 =$

$$\begin{bmatrix} 0 & 6 & 7 & 10 & 13 & 15 & 10 & 19 & 20 & 18 \\ 6 & 0 & 12 & 16 & 8 & 20 & 15 & 17 & 25 & 23 \\ 7 & 12 & 0 & 4 & 6 & 8 & 3 & 12 & 13 & 11 \\ 10 & 16 & 4 & 0 & 10 & 12 & 7 & 16 & 17 & 15 \\ 13 & 8 & 6 & 10 & 0 & 13 & 9 & 9 & 19 & 16 \\ 15 & 20 & 8 & 12 & 13 & 0 & 5 & 4 & 8 & 3 \\ 10 & 15 & 3 & 7 & 9 & 5 & 0 & 9 & 10 & 8 \\ 19 & 17 & 12 & 16 & 9 & 4 & 9 & 0 & 12 & 7 \\ 20 & 25 & 13 & 17 & 19 & 8 & 10 & 12 & 0 & 9 \\ 18 & 23 & 11 & 15 & 16 & 3 & 8 & 7 & 9 & 0 \end{bmatrix}$$

Above matrix is optimal matrix and entries are the shortest distance of network given in figure 1.

Figure 2: Road Map of Sindh (source: Highway Department)

4.2 Network and its distances

Road network data of Sindh is considered here. This real road network, have main towns and five level of roads given in [6]

- motorways (R-1)
- national highways (R-2)
- metalledmain (R-3),
- metalled other(R-5) and
- unmetalled (R-5) roads

This road map(figure 2) contains 17 main cities of Sindh province, which means network contains 17 vertices or nodes are listed below,

1) Karachi
2) Thatta
3) Badin
4) Mithi
5) Hyderabad
6) Mirpurkhas
7) Umarkot
8) Sanghar
9) Nawabshah
10) Nusheroferoz
11) Dadu
12) Larkana
13) Kherpur
14) Sukkar
15) Shikarpur
16) Jacobabad
17) Ghotki

As much as the sources of information are concerned, all the distances of roads are taken from [6], Surveyor general of Pakistan or estimated distances has calculated directly from the road map where ever required [6]. The distances of seventeen main cities of Sindh are mentioned in the list in table1.

Table 1. Edge-wise Approximate Distances in Road Network of Sindh

Edge no	From	To	Total distance	Route
1	Karachi	Thatha	101	Karachi-Thata
2	Karachi	Hyderabad	175	Karachi-Kotri-Hyderabad
3	Karachi	Dadu	340	Karachi-Kotri-Sehwan-Dadu
4	Thatta	Badin	93	Thatta-Sujawal-Badin
5	Thatta	Hyderabad	98	Thatta-Hyderabad
6	Badin	Mithi	110	Badin-TandoBago-Jhudo-Naukot-Mithi
7	Badin	Hyderabad	100	Badin-Matli-Hyderabad
8	Badin	MirpurKhas	138	Badin-Matli-DigriMirwah-MirpurKhas
9	Badin	Umarkot	175	Badin-Matli-DigriMirwah-Umarkot
10	Mithi	Hyderabad	154	Mithi-Jhudo-Digri-ShaikhBhirklo-Hyderabad
11	Mithi	MirpurKhas	120	Mithi-Jhudo-Mirwah-MirpurKhas
12	Mithi	Umarkot	115	Mithi-Jhudo-Nabisar-Umarkot
13	Hyderabad	Mirpurkhas	66	Hyderabad-TandoAllahyar-MirpurKhas
14	Hyderabad	Sanghar	104	Hyderabad-Hala-Shahdadpur-Sanghar
15	Hyderabad	Nawabshah	113	Hyderabad-Hala-Sakrand-Nawabshah
16	Hyderabad	NausharoFiroz	182	Hyderabad-Sakrand-Moro-NaushahroFiroz
17	Hyderabad	Dadu	180	Hyderabad-Kotri-Sehwan-Dadu
18	MirpurKhas	Umarkot	72	MirpurKhas-Pithoro-Umarkot
19	Mirpurkhas	Sanghar	63	MirpurKhas-Sanghar
20	Sanghar	Nawabshah	61	Sanghar-Khadro-Nawabshah
21	Nawabshah	NaushahroFiroz	80	Nawabshah-Pad Idan-NaushahroFiroz
22	Nawabshah	Dadu	114	Nawabshah-Sakrand-Moro-Dadu
23	Nawabshah	Khairpur	143	Nawabshah-Pad Idan-KotDiji-Khairpur
24	NausharaFiroz	Dadu	48	NaushahroFiroz-Moro-Dadu
25	NaushahroFiroz	Khairpur	113	NaushahroFiroz-Ranjpur-Khairpur
26	Dadu	Larkana	110	Dadu-Mehar-Larkana
27	Larkana	Shikarpur	63	Larkana-GarhiYasin-Shikarpur
28	Larkana	Jacobabad	100	Larkana-Ratodero-Jacobabad
29	Khairpur	Sukkur	23	Khairpur-Sukkur
30	Sukkar	Shikarpur	37	Sukkur-Shikarpur
31	Sukkar	Ghotki	58	Sukkur-PanoAqil-Ghotki
32	Shikarpur	Jacobabad	43	Shikarpur-Jacobabad

By using figure 2 (Sindh route map) and table 1(total distance) simple undirected graph is drawn, where vertices or nodes represents seventeen cities and edges represents total distance.

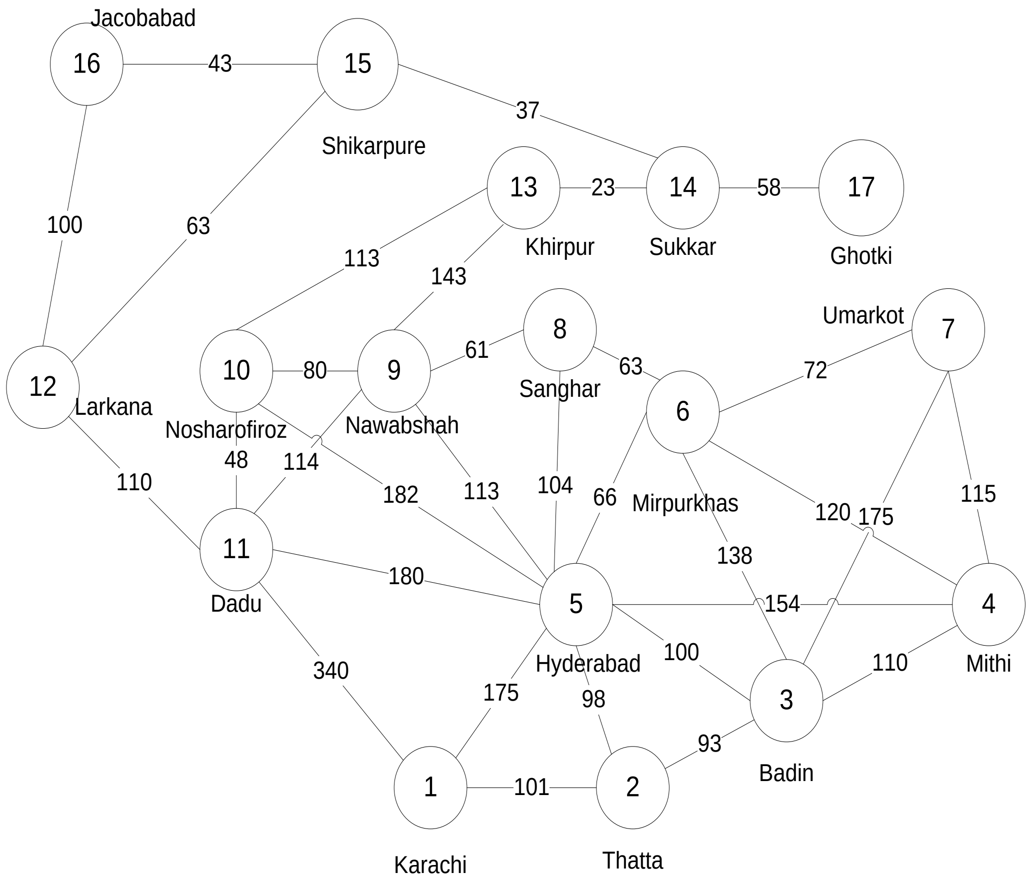

Figure 3: Weighted Graph of Road Network in Fig 2

Using structural matrix definition 3.2 a matrix is formed

$$IM=\begin{bmatrix}
0 & 101 & \infty & \infty & 175 & \infty & \infty & \infty & \infty & \infty & 340 & \infty & \infty & \infty & \infty & \infty & \infty \\
101 & 0 & 93 & \infty & 98 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & 93 & 0 & 110 & 100 & 138 & 175 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 110 & 0 & 154 & 120 & 115 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
175 & 98 & 100 & 154 & 0 & 66 & \infty & 104 & 113 & 182 & 180 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 138 & 120 & 66 & 0 & 72 & 63 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 175 & 115 & \infty & 72 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 104 & 63 & \infty & 0 & 61 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 113 & \infty & \infty & 61 & 0 & 80 & 114 & \infty & 143 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 182 & \infty & \infty & \infty & 80 & 0 & 48 & \infty & 113 & \infty & \infty & \infty & \infty \\
340 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 114 & 48 & 0 & 110 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 110 & 0 & \infty & \infty & 63 & 100 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 143 & 113 & \infty & \infty & 0 & 23 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 23 & 0 & 37 & \infty & 58 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 63 & \infty & 37 & 0 & 43 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 100 & \infty & \infty & 43 & 0 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 58 & \infty & \infty & 0
\end{bmatrix}$$

This square matrix of order 17x17 is **initial matrix (IM)**, we applied Min-sum Algebra by using MATLAB which can perform matrix manipulations, plotting of data and functions, algorithms implementation, interfacing with programs written in other languages and creation of user interfaces[11] to find the optimal matrix(OM).

Using Matlab, develop the function file of any particular name e.g. 'Min-sum algebra' and write the code to perform Min-sum Algebra of a square matrix.

After saving matlab function file we write initial matrix given (which we developed from figure 3 on command window of matlab, and then write 'Min-sum Algebra(A) (matlab file name) on command window of matlab to get the final result. Optimal matrix below shows the shortest distance of road networks for Sindh mentioned in figure 1.

$$OM = \begin{bmatrix} 0 & 101 & 194 & 304 & 175 & 241 & 313 & 279 & 288 & 357 & 340 & 450 & 431 & 454 & 491 & 534 & 512 \\ 101 & 0 & 93 & 203 & 98 & 164 & 236 & 202 & 211 & 280 & 278 & 388 & 354 & 377 & 414 & 457 & 435 \\ 194 & 93 & 0 & 110 & 100 & 138 & 175 & 201 & 213 & 282 & 280 & 390 & 356 & 379 & 416 & 459 & 437 \\ 304 & 203 & 110 & 0 & 154 & 120 & 115 & 183 & 244 & 324 & 334 & 444 & 387 & 410 & 447 & 490 & 468 \\ 175 & 98 & 100 & 154 & 0 & 66 & \infty & 104 & 113 & 182 & 180 & 290 & 256 & 279 & 316 & 359 & 337 \\ 241 & 164 & 138 & 120 & 66 & 0 & 72 & 63 & 124 & 204 & 238 & 348 & 267 & 290 & 327 & 370 & 348 \\ 313 & 236 & 175 & 115 & 138 & 72 & 0 & 135 & 196 & 276 & 310 & 420 & 339 & 362 & 399 & 442 & 420 \\ 279 & 202 & 201 & 183 & 104 & 63 & 135 & 0 & 61 & 141 & 175 & 285 & 204 & 227 & 264 & 307 & 285 \\ 288 & 211 & 213 & 244 & 113 & 124 & 196 & 61 & 0 & 80 & 114 & 224 & 143 & 166 & 203 & 246 & 224 \\ 357 & 280 & 282 & 324 & 182 & 204 & 276 & 141 & 80 & 0 & 48 & 158 & 113 & 136 & 173 & 216 & 194 \\ 340 & 278 & 280 & 334 & 180 & 238 & 310 & 175 & 114 & 48 & 0 & 110 & 161 & 184 & 173 & 210 & 242 \\ 450 & 388 & 390 & 444 & 290 & 348 & 420 & 285 & 224 & 158 & 110 & 0 & 123 & 100 & 63 & 100 & 158 \\ 431 & 354 & 356 & 387 & 256 & 267 & 339 & 204 & 143 & 113 & 161 & 123 & 0 & 23 & 60 & 103 & 81 \\ 454 & 377 & 379 & 410 & 279 & 279 & 362 & 227 & 166 & 136 & 184 & 100 & 23 & 0 & 37 & 80 & 58 \\ 491 & 414 & 416 & 447 & 316 & 316 & 399 & 264 & 203 & 173 & 173 & 63 & 60 & 37 & 0 & 43 & 95 \\ 534 & 457 & 459 & 490 & 359 & 359 & 442 & 307 & 246 & 216 & 210 & 100 & 103 & 80 & 43 & 0 & 138 \\ 512 & 435 & 437 & 468 & 337 & 337 & 420 & 285 & 224 & 194 & 242 & 158 & 81 & 58 & 95 & 138 & 0 \end{bmatrix}$$

4.3 Finding Optimal Path by forward and backward tracking

Now the main objective of this study is to apply forward and backward tracking with the help of initial and optimal matrices of above mentioned Min-sum Algebra and to find the path or route of shortest distance.

4.3.1 Forward Tracking

To apply forward tracking, any two vertices or nodes are selected and one vertex is assigned as source and other one as destination.

Consider node 1 or V1 as source and node 16 or V16 as destination in IM(initial matrix).

1. Select value of V16 from optimal matrix.OM=>The minimum distance from V1 to V16 is 534.
2. Select any suitable non zero numeric value from the column of V6 in Initial matrix (IM).A=> There are only two non-zero numeric vertices or nodes we have, V12 = 100 and V15 = 43. Suitable one is V15 because 534-43=491, and in the row of OM V15 = 491, V12 is not satisfying the OM solution (If two vertices or nodes satisfying the optimal matrix then select the minimum distance).
3. Again select any suitable non zero numeric value from the column of V15,now we have three non-zero numeric values on V15 which are 63,37 and 43,37 is suitable because it is satisfying V14 of OM.

Repeat the above mentioned steps 3 till V1 as shown in Figure 4.

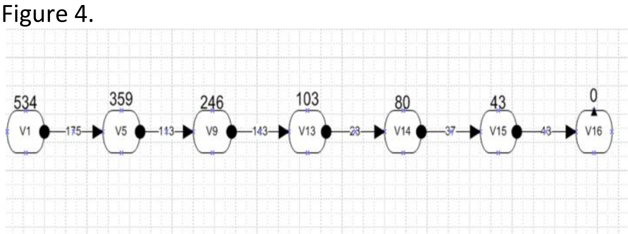

Figure 4: Forward tracking from V1 to V16

Figure 4 shows the shortest route followed by Min-sum Algebra to get the shortest distance. By using this forward tracking method we can find the route of any vertices or nodes.

4.3.2 Backward Tracking

Backward Tracking is quite similar to forward tracking, to apply backward tracking any two vertices or nodes are selected and one vertex is assigned as source and other one as destination from above mentioned road network. Consider vertex 1 or V1 as source and vertex 16 or V16 as destination defined in IM(initial matrix).

1. Select value of V16 from optimal matrix.OM=>The minimum distance from V1 to V16 is 534.
2. Select any suitable non-zero numeric value from the column of V1 in Initial matrix(IM).A=> There are only three non-zero numeric vertices or nodes we have, V5 = 175,V2 = 101 and V11 = 340. Suitable one is V5 because 534-175=359, and in the column of OM V5 = 559, V2 and V11 are not satisfying the OM solution (If two vertices or nodes are satisfying the optimal matrix then select the minimum distance).
3. Again select any suitable non zero numeric value from the column of V5,now there are nine non zero numeric values in the column of V5 are 175,98,100,154,66,104,113,182 and 180.but 113 is suitable because it is satisfying column of V9 of OM.

Repeat the above mentioned steps 3 till V16 as shown in Figure 5.

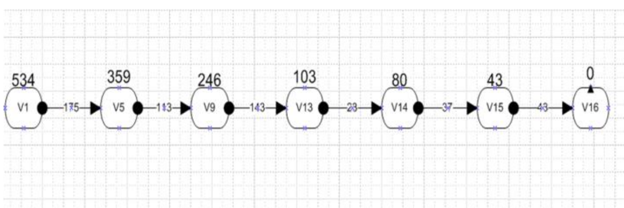

Figure 5: Forward tracking from V1 to V16

Figure 5 shows the shortest route followed by Min-sum Algebra to get the shortest distance.
By using this backward tracking method we can find the route of any vertices or nodes.

4.3.3 Comparison between forward and backward tracking

If ending and starting nodes are same, forward and backward tracking gives the same path. In this paper both ways tracking gives same path as mentioned in fig 4 and fig 5.

Conclusion

Nowadays, we have observed an increasing popularity of transportation related decision analysis, In this type of analysis, the calculation of shortest paths is often a central task because the shortest path distances are often required as input for "higher level" models in many transportation analysis problems such as network flows, facility location, product delivery and vehicle routing, etc.. As a concern, these analysis tasks demand high performance shortest path algorithms that run fastest on real road networks. All existing shortest path problems are an overview of the traditional problems of shortest path. These are, optimal path connecting and searching an initial node or vertex with a target node or vertex, passing through a middle node set, involving each other by non-directed path, from which, length, distance, average altitude, transportation costs, time attributes, etc., are well-known. All well-known shortest path algorithms allow an initial path population by random generation. Each path-individual has a condition function facilitating to be differentiated with others, and then through genetic operators contribute in next generations development of better paths, calculating each time better paths for the trajectories with the best fitness from the previous population while The Modified Min-sum Algebra method calculates the shortest distance, length, time and cost etc. directly of any network and it also defines the shortest route as well, which followed by a network to calculate the shortest distances, time costs or length etc.

Acknowledgments

We acknowledged the all kind of support by NED University of Engineering & Technology and I would also like to express my sincere gratitude to my best friend, Noor Fatima Siddiqui whose expertise, motivation, understanding, encouragement and patience, added considerable efforts to Matlab code

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