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# Connected Total Dominating Sets and Connected Total Domination Polynomials of Triangular Ladders

**A. Vijayan[1] and T. Anitha Baby[2]**

[1]Associate Professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Tamil Nadu, India
[2]Assistant Professor, Department of Mathematics, Women's Christian College, Nagercoil, Tamil Nadu, India

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***Abstract***

*Let G be a simple connected graph of order n. Let $D_{ct}(G, i)$ be the family of connected total dominating sets of G with cardinality i. The polynomial $D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^{n} d_{ct}(G, i)\, x^i$ is called the connected total domination polynomial of G. In this paper, we study some properties of connected total domination polynomials of the Triangular Ladder $TL_n$. We obtain a recursive formula for $d_{ct}(TL_n, i)$. Using this recursive formula, we construct the connected total domination polynomial $D_{ct}(TL_n, x) = \sum_{i=n-1}^{2n} d_{ct}(TL_n, i)\, x^i$, of $TL_n$, where $d_{ct}(TL_n, i)$ is the number of connected total dominating sets of $TL_n$ with cardinality i and some properties of this polynomial have been studied.*

***Keywords**: Triangular Ladder, connected total dominating set, connected total domination number, connected total domination polynomial.*

## 1. Introduction

Let G = (V, E) be a simple connected graph of order n. For any vertex v∈V, the open neighbourhood of v is the set N(v) = {u∈V/uv∈E} and the closed neighbourhood of v is the set N[v] = N(v) ∪ {v}. For a set S ⊆ V, the open neighbourhood of S is $N(S) = \bigcup_{v\in S} N(v)$ and the closed neighbourhood of S is N[S] = N(S) ∪ S. The maximum degree of the graph G is denoted by Δ(G) and the minimum degree is denoted by δ(G).

A set S of vertices in a graph G is said to be a total dominating set if every vertex v∈V is adjacent to an element of S.

A total dominating set S of G is called a connected total dominating set if the induced subgraph ⟨S⟩ is connected.

The minimum cardinality taken over all connected total dominating sets S of G is called the connected total domination number of G and is denoted by $\gamma_{ct}(G)$.

A connected total dominating set with cardinality $\gamma_{ct}(G)$ is called $\gamma_{ct}$ – set. We denote the set {1, 2, . . ., 2n –1, 2n} by [2n], throughout this paper.

## 2. Connected total dominating sets of triangular ladders

Consider two paths $[u_1u_2\ldots u_n]$ and $[v_1v_2\ldots v_n]$. Join each pair of vertices $u_i,v_i$ and $u_{i+1},v_i$, i = 1, 2,…, n. The resulting graph is a Triangular Ladder.

Let $TL_n$ be a Triangular ladder with 2n vertices. Label the vertices of $TL_n$ as given in Figure 1.

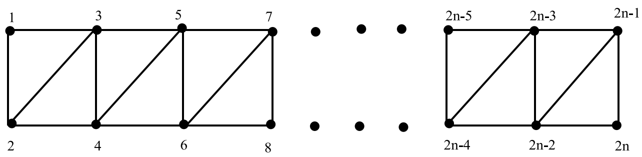


**Figure 1**

**Triangular Ladder $TL_n$**

Then, $V(TL_n)$ = {1,2, 3, …, 2n – 3, 2n–2, 2n – 1, 2n} and $E(TL_n)$ = {(1,3), (3,5), (5,7), …, (2n–5, 2n–3), (2n–3, 2n–1), (2, 4), (4, 6), (6, 8),…, (2n–4, 2n–2), (2n–2, 2n), (1, 2), (3, 4), (5, 6), …, (2n–3, 2n–2), (2n–1, 2n),(2, 3), (4, 5), (6, 7),…,(2n–4, 2n–3),(2n–2, 2n–1)}.

For the construction of the connected total dominating sets of the Triangular Ladders $TL_n$, we need to investigate the connected total dominating sets of $TL_n$ – {2n}. In this section, we investigate the connected total dominating sets of $TL_n$ with cardinality i. We shall find the recursive formula for $d_{ct}(TL_n, i)$.

**Lemma 2.1[7].**

$\gamma_{ct}(P_n) = n - 2$.

**Lemma 2.2**

For every $n \in N$ and $n \geq 4$,

(i) $\gamma_{ct}(TL_n) = n - 1$.

(ii) $\gamma_{ct}(TL_n - \{2n\}) = n - 2$.

(iii) $D_{ct}(TL_n, i) = \phi$ if and only if $i < n - 1$ or $i > 2n$.

(iv) $D_{ct}(TL_n - \{2n\}, i) = \phi$ if and only if $i < n - 2$ or $i > 2n - 1$.

**Proof**

(i) Clearly {3,5,7,9,…,2n −1} is a minimum connected total dominating set for $TL_n$. If n is even or odd it contains n – 1 elements. Hence, $\gamma_{ct}(TL_n) = n - 1$.

(ii) Clearly { 3,5,7,9,…, 2n −3} is a minimum connected total dominating set for $TL_n - \{2n\}$. If n is even or odd it contains n – 2 elements. Hence, $\gamma_{ct}(TL_n - \{2n\}) = n - 2$.

(iii) follows from (i) and the definition of connected total dominating set.

(iv) follows from (ii) and the definition of connected total dominating set.

**Lemma 2.3**

(i) If $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$ and $D_{ct}(TL_{n-2}, i - 1) = \phi$, then $D_{ct}(TL_{n-1}, i - 1) = \phi$.

(ii) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-2}, i - 1) \neq \phi$, then $D_{ct}(TL_{n-1}, i - 1) \neq \phi$.

(iii) If $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$ and $D_{ct}(TL_{n-1}, i - 1) = \phi$, then

$D_{ct}(TL_n, i) = \phi$.

(iv) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-1}, i - 1) \neq \phi$, then

$D_{ct}(TL_n, i) \neq \phi$.

(v) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, and $D_{ct}(TL_{n-1}, i - 1) = \phi$, then

$D_{ct}(TL_n, i) \neq \phi$.

**Proof**

(i) Since, $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$,
$D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$ and
$D_{ct}(TL_{n-2}, i - 1) = \phi$, by Lemma 2.2 (iii) & (iv), we have,
$i - 1 < n - 2$ or $i - 1 > 2n - 1$,
$i - 1 < n - 3$ or $i - 1 > 2n - 3$ and
$i - 1 < n - 3$ or $i - 1 > 2n - 4$.
Therefore, $i - 1 < n - 3$ or $i - 1 > 2n - 1$.
Therefore, $i - 1 < n - 2$ or $i - 1 > 2n - 2$ holds.
Hence, $D_{ct}(TL_{n-1}, i - 1) = \phi$.

(ii) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$,
$D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) \neq \phi$ and
$D_{ct}(TL_{n-2}, i - 1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$,
$n - 3 \leq i - 1 \leq 2n - 3$ and
$n - 3 \leq i - 1 \leq 2n - 4$.
Suppose, $D_{ct}(TL_{n-1}, i - 1) = \phi$.
Then, by Lemma 2.2 (iii), we have, $i - 1 < n - 2$ or $i - 1 > 2n - 2$.
If $i - 1 < n - 2$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction.
If $i - 1 > 2n - 2$, then $i - 1 > 2n - 3$ holds, which implies $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$, a contradiction.
Therefore, $D_{ct}(TL_{n-1}, i - 1) \neq \phi$.

(iii) Since, $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$ and $D_{ct}(TL_{n-1}, i - 1) = \phi$,

by Lemma 2.2 (iii) & (iv), we have ,

$i - 1 < n - 2$ or $i - 1 > 2n - 1$ and

$i - 1 < n - 2$ or $i - 1 > 2n - 2$.

Therefore, $i - 1 < n - 2$ or $I - 1 > 2n - 1$.

Therefore, $i < n - 1$ or $i > 2n$.

Hence, $D_{ct}(TL_n, i) = \phi$.

(iv) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-1}, i - 1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$ and
$n - 2 \leq i - 1 \leq 2n - 2$.
Suppose, $D_{ct}(TL_n, i) = \phi$, then, by Lemma 2.2 (iii),
we have $i < n - 1$ or $i > 2n$.
Therefore , $i - 1 < n - 2$ or $i - 1 > 2n - 1$.
If $i - 1 < n - 2$, then $D_{ct}(TL_{n-1}, i - 1) = \phi$, a contradiction.
If $i - 1 > 2n - 1$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction.
Therefore, $D_{ct}(TL_n, i) \neq \phi$.

(v) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, by Lemma 2.2 (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$.
Also, since, $D_{ct}(TL_{n-1}, i - 1) = \phi$, by Lemma 2.2 (iii), we have,
$i - 1 < n - 2$ or $i - 1 > 2n - 2$.
If $i - 1 < n - 2$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction.
Therefore, $i - 1 > 2n - 2$.
Therefore, $i - 1 \geq 2n - 1$.

Also, $i - 1 \leq 2n - 1$.
Therefore, $i - 1 = 2n - 1$.
Therefore, $i = 2n$.
Hence, $D_{ct}(TL_n, i) \neq \phi$.

**Lemma 2.4**

Suppose that $D_{ct}(TL_n, i) \neq \phi$, then for every $n \in N$,

(i) $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) = \phi$ if and only if $i = 2n$.

(ii) $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) = \phi$, if and only if $i = 2n - 1$.

(iii) $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$, $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-2}, i-1) = \phi$ if and only if $i = 2n - 2$.

**Proof**

Assume that, $D_{ct}(TL_n, i) \neq \phi$.

Therefore, $n - 1 \leq i \leq 2n$.

(i) ($\Rightarrow$) since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, by Lemma 2.2 (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$.
Therefore, $n - 1 \leq i \leq 2n$.
Also, since, $D_{ct}(TL_{n-1}, i-1) = \phi$, by Lemma 2.2 (iii), we have,
$i - 1 < n - 2$ or $i - 1 > 2n - 2$.
If $i - 1 < n - 2$, then $i < n - 1$ which implies $D_{ct}(TL_n, i) = \phi$, a contradiction.
Therefore, $i - 1 > 2n - 2$.
Therefore, $i > 2n - 1$.
Therefore, $i \geq 2n$.
Together we have, $i = 2n$.
($\Leftarrow$) follows from Lemma 2.2 (iii) & (iv).

(ii) ($\Rightarrow$) Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, and $D_{ct}(TL_{n-1}, i-1) \neq \phi$,
by Lemma 2.2 (iii) & (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$ and
$n - 2 \leq i - 1 \leq 2n - 2$.
Therefore, $n - 2 \leq i - 1 \leq 2n - 2$.
Therefore, $n - 1 \leq i \leq 2n - 1$.
Also, since, $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) = \phi$,
by Lemma 2.2 (iv), we have, $i - 1 < n - 3$ or $i - 1 > 2n-3$.
Therefore, $i < n - 2$ or $i > 2n - 2$.
If $i < n - 2$, the $i < n - 1$ holds, which implies $D_{ct}(TL_n, i) = \phi$, a contradiction.
Therefore, $i > 2n - 2$.
Therefore, $i \geq 2n - 1$.
Together we have, $i = 2n - 1$.
($\Leftarrow$) follows from Lemma 2.2 (iii) & (iv).

(iii)($\Rightarrow$) Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,
$n - 2 \leq i - 1 \leq 2n - 1$,
$n - 2 \leq i - 1 \leq 2n - 2$ and
$n - 3 \leq i - 1 \leq 2n - 3$.
Therefore, $n - 2 \leq i - 1 \leq 2n - 3$.
Therefore, $n - 1 \leq i \leq 2n - 2$.
Also, since, $D_{ct}(TL_{n-2}, i-1) = \phi$, by Lemma 2.2 (iii), we have, $i - 1 < n - 3$, or $i - 1 > 2n - 4$.
If $i - 1 < n - 3$, then $i < n - 2$.
Therefore, $i < n - 1$ holds, which implies $D_{ct}(TL_n, i) = \phi$, a contradiction.
Therefore, $i - 1 > 2n - 4$.
Therefore, $i > 2n - 3$.
Therefore, $i \geq 2n - 2$.
Together we have $i = 2n - 2$.
($\Leftarrow$) follows from Lemma 2.2 (iii) & (iv).

**Theorem 2.5**

For every $n \geq 4$,
(i) If $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) = \phi$, then $D_{ct}(TL_n, i) = \{X \cup \{2n\} / X \in D_{ct}(TL_n - \{2n\}, i-1)\}$.
(ii) If $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) \neq \phi$, then
$D_{ct}(TL_n, i) = \{X_1 \cup \{2n-1\}$, if $2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}$, if $2n-2$ or $2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}$, if $2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.

**Proof**

(i) Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) = \phi$, by Theorem 2.4 (i), $i = 2n$.
Therefore, $D_{ct}(TL_n, i) = D_{ct}(TL_n, 2n) = \{[2n]\}$ and
$D_{ct}(TL_n - \{2n\}, i-1) = D_{ct}(TL_n - \{2_n\}, 2n-1) = \{[2n-1]\}$, we have the result.
(ii) Let $Y_1 = \{X_1 \cup \{2n-1\}$, if $2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}$, if $2n-2$ or $2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\}$
and $Y_2 = \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}$, if $2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.
Obviously, $Y_1 \cup Y_2 \subseteq D_{ct}(TL_n, i)$ (1)

Now, let $Y \in D_{ct}(TL_n, i)$.
If $2n \in Y$, then atleast one of the vertices labeled $2n-2$ or $2n-1$ is in Y. In either cases, $Y = \{X_1 \cup \{2n\}\}$ for some $X_1 \in D_{ct}(TL_n - \{2n\}, i-1)$.
Therefore, $Y \in Y_1$.

Suppose that, $2n-1 \in Y$, $2n \notin Y$, then atleast one of the vertices labeled $2n-3$ or $2n-2$ is in Y.

If $2n-3 \in Y$, then $Y = \{X_1 \cup \{2n-1\}\}$ for some $X_1 \in D_{ct}(TL_n - \{2n\}, i-1)$.

If $2n-2 \in Y$, then $Y = \{X_2 \cup \{2n-1\}\}$ for some $X_2 \in D_{ct}(TL_{n-1}, i-1)$.

Therefore, $Y \in Y_1$ or $Y \in Y_2$.

Now, Suppose that,

$2n-2 \in Y$, $2n-1 \notin Y$, $2n \notin Y$, then atleast one of the vertices labeled $2n-4$ or $2n-3$ is in Y.

In either cases, $Y = \{X_2 \cup \{2n-2\}\}$ for some $X_2 \in D_{ct}(TL_{n-1}, i-1)$.

Therefore, $Y \in Y_2$.

Hence, $D_{ct}(TL_n, i) \subseteq Y_1 \cup Y_2$ (2)

From (1) and (2), we have,

$D_{ct}(TL_n, i) = \{X_1 \cup \{2n-1\}$, if $2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}$, if $2n-2$ or $2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}$, if $2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.

**Theorem 2.6**

If $D_{ct}(TL_n, i)$ be the family of connected total dominating sets of $TL_n$ with cardinality i, where $i \geq n-1$, then $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i-1) + dct(TL_{n-1}, i-1)$.

**Proof**

We consider the two cases given in Theorem 2.5.

By Theorem 2.5 (i), we have,

$D_{ct}(TL_n, i) = \{X \cup \{2n\} / X \in D_{ct}(TL_n - \{2n\}, i-1)\}$.

Since, $D_{ct}(TL_{n-1}, i-1) = \phi$, we have,

$dct(TL_{n-1}, i-1) = 0$.

Therefore, $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i-1)$.

By Theorem 2.5 (ii), we have,

$D_{ct}(TL_n, i) = \{X_1 \cup \{2n-1\}$, if $2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}$, if $2n-2$ or $2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}$, if $2n-4$ or $2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}$, if $2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.

Therefore,

$d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i-1) + d_{ct}(TL_{n-1}, i-1)$.

## 3. Connected total domination polynomials of triangular ladders

**Definition 3.1**

Let $D_{ct}(TL_n, i)$ be the family of connected total dominating sets of $TL_n$ with cardinality i and let $d_{ct}(TL_n, i) = |D_{ct}(TL_n, i)|$. Then the connected total domination Ploynomial $D_{ct}(TL_n, x)$ of $TL_n$ is defined as,

$$D_{ct}(TL_n, x) = \sum_{i=\gamma_{ct}(TL_n)}^{2n} d_{ct}(TL_n, i)\, x^i.$$

**Theorem 3.2**

For every $n \geq 4$,

$D_{ct}(TL_n, x) = x[D_{ct}(TL_n - \{2n\}, x) + D_{ct}(TL_{n-1}, x)]$, with initial values,

$D_{ct}(TL_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(TL_2, x) = 5x^2 + 4x^3 + x^4$.

$D_{ct}(TL_3 - \{6\}, x) = 5x^2 + 8x^3 + 5x^4 + x^5$.

$D_{ct}(TL_3, x) = 3x^2 + 10x^3 + 12x^4 + 6x^5 + x^6$.

$D_{ct}(TL_4 - \{8\}, x) = x^2 + 8x^3 + 18x^4 + 17x^5 + 7x^6 + x^7$.

**Proof**

We have, $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i-1) + d_{ct}(TL_{n-1}, i-1)$.

Therefore, $d_{ct}(TL_n, i)\, x^i = d_{ct}(TL_n - \{2n\}, i-1)\, x^i + d_{ct}(TL_{n-1}, i-1)\, x^i$.

$\Sigma d_{ct}(TL_n, i)\, x^i = \Sigma d_{ct}(TL_n - \{2n\}, i-1)\, x^i + \Sigma d_{ct}(TL_{n-1}, i-1)\, x^i$.

$\Sigma d_{ct}(TL_n, i)\, x^i = x \Sigma d_{ct}(TL_n - \{2n\}, i-1)\, x^{i-1} + x \Sigma d_{ct}(TL_{n-1}, i-1) x^{i-1}$.

$D_{ct}(TL_n, x) = x\, D_{ct}(TL_n - \{2n\}, x) + x\, D_{ct}(TL_{n-1}, x)$.

Therefore, $D_{ct}(TL_n, x) = x[D_{ct}(TL_n - \{2n\}, x) + D_{ct}(TL_{n-1}, x)]$ with initial values,

$D_{ct}(TL_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(TL_2, x) = 5x^2 + 4x^3 + x^4$.

$D_{ct}(TL_3 - \{6\}, x) = 5x^2 + 8x^3 + 5x^2 + x^5$.

$D_{ct}(TL_3, x) = 3x^2 + 10x^3 + 12x^4 + 6x^5 + x^6$.

$D_{ct}(TL_4 - \{8\}, x) = x^2 + 8x^3 + 18x^4 + 17x^5 + 7x^6 + x^7$.

**Example 3.3**

$D_{ct}(TL_4, x) = 4x^3 + 18x^4 + 30x^5 + 23x^6 + 8x^7 + x^8$.

$D_{ct}(TL_5 - \{10\}, x) = x^3 + 12x^4 + 36x^5 + 47x^6 + 30x^7 + 9x^8 + x^9$.

By Theorem 3.2, we have,

$D_{ct}(TL_5, x) = x[4x^3 + 18x^4 + 30x^5 + 23x^6 + 8x^7 + x^8 + x^3 + 12x^4 + 36x^5 + 47x^6 + 30x^7 + 9x^8 + x^9]$.

$= 5x^4 + 30x^5 + 66x^6 + 70x^7 + 38x^8 + 10x^9 + x^{10}$.

We obtain $d_{ct}(TL_n, i)$ and $d_{ct}(TL_n - \{2n\}, i)$ for $2 \leq n \leq 9$ as shown in Table 1.

**Table 1**

| n \ i | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $TL_2$-{4} | 3 | 1 | | | | | | | | | | | | | | | |
| $TL_2$ | 5 | 4 | 1 | | | | | | | | | | | | | | |
| $TL_3$-{6} | 5 | 8 | 5 | 1 | | | | | | | | | | | | | |
| $TL_3$ | 3 | 10 | 12 | 6 | 1 | | | | | | | | | | | | |
| $TL_4$-{8} | 1 | 8 | 18 | 17 | 7 | 1 | | | | | | | | | | | |
| $TL_4$ | 0 | 4 | 18 | 30 | 23 | 8 | 1 | | | | | | | | | | |
| $TL_5$-{10} | 0 | 1 | 12 | 36 | 47 | 30 | 9 | 1 | | | | | | | | | |
| $TL_5$ | 0 | 0 | 5 | 30 | 66 | 70 | 38 | 10 | 1 | | | | | | | | |
| $TL_6$-{12} | 0 | 0 | 1 | 17 | 66 | 113 | 100 | 47 | 11 | 1 | | | | | | | |
| $TL_6$ | 0 | 0 | 0 | 6 | 47 | 132 | 183 | 138 | 57 | 12 | 1 | | | | | | |
| $TL_7$-{14} | 0 | 0 | 0 | 1 | 23 | 113 | 245 | 283 | 185 | 68 | 13 | 1 | | | | | |
| $TL_7$ | 0 | 0 | 0 | 0 | 7 | 70 | 245 | 428 | 421 | 242 | 80 | 14 | 1 | | | | |
| $TL_8$-{16} | 0 | 0 | 0 | 0 | 1 | 30 | 183 | 490 | 711 | 606 | 310 | 93 | 15 | 1 | | | |
| $TL_8$ | 0 | 0 | 0 | 0 | 0 | 8 | 37 | 428 | 918 | 1132 | 848 | 390 | 107 | 16 | 1 | | |
| $TL_9$-{18} | 0 | 0 | 0 | 0 | 0 | 1 | 38 | 220 | 918 | 1629 | 1738 | 1158 | 483 | 122 | 17 | 1 | |
| $TL_9$ | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 75 | 648 | 1836 | 2761 | 2586 | 1548 | 590 | 138 | 18 | 1 |

In the following Theorem we obtain some properties of $d_{ct}(TL_n, i)$.

**Theorem 3.4**

The following properties hold for the coefficients of $D_{ct}(TL_n, x)$ for all n.

(i) $d_{ct}(TL_n, 2n) = 1$, for every $n \geq 2$.
(ii) $d_{ct}(TL_n, 2n - 1) = 2n$, for every $n \geq 2$.
(iii) $d_{ct}(TL_n, 2n - 2) = 2n^2 - 3n + 3$, for every $n \geq 2$.
(iv) $d_{ct}(TL_n\ n - 1) = n$, for every $n \geq 3$.

**Proof**

Proof is obvious.

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