

Fuzzy Analysis of Particle movement in Quantum Mechanics

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Abstract

Direct measurement of variables in classic mechanics is not applicable to whole of real-deterministic areas like non-linear functions of quantum mechanics; and corporation of beneficent methods like fuzzy logic can predict the behavior of particles in the fields that, there is not much access for experience of all possible states in physics. Thus, based on Schrödinger equation and momentum of particles in a 2-D surface with a wave-like function, the hidden aspects of behavior is forecasted and for any given state as input, fuzzy controller indicates the characteristics as well.

Keywords: Fuzzy logic, Schrodinger equation, Particle momentum, Relative velocity

Introduction

The concepts of fuzzy logic and quantum mechanics could be brought together in analysis of particles behavior in the material. Demonstrating of fuzzy analysis while interpretation of the Shrodinger equation is advantageous at passage from classic to quantum mechanics. The basic ways of fuzziness are proposed by Kosko (1993), who wrote the required principles at survey over a matter of degree in related issues. A merit of fuzzy analysis is inexistence of boundary in transformation of knowledge from classical to quantum mechanics and in this regard, assigning of crisp ratings has been found too helpful for prediction of behaviors in quantum mechanics. The greatest disadvantage which is mentionable in classical mechanics is hidden facts of a single measurement and applying of various methods (such as averaging) for enhancement of results certainty level will be useful.

Thus one of the famous methods known in this way is fuzzy analysis, which by calculating in related platform and introduction of basic rules, whole of other possible behaviors are predictable and a comprehensive supervision over mechanics of particles could provide a better analysis of unpredictable situations and levels of reliability is improved as well. Our basic assumption is that reality is found in fuzzy products and non-locality in space and time are subjects which should be indicated. So it is quite reasonable that by connecting the underlying traces of classical theory to quantum mechanics, in the bottom-up view of particles to material configuration, the interconnecting bridges offer accessibility of critical analysis, compared to ordinary sense thinking.

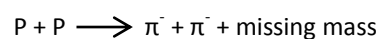
Literature Review

Previous literature about survey over quantum correlation of particles is shown in the Table 1 as follows:

Table1. Previous researches in Particles mechanics

Literature	Subject
Biales <i>et al</i> (1972)	Particle entropy formulation
Heyot & Moral (1974)	Intersecting Storage Rings (ISR)
Moral & Plaut (1974)	Multiplicity of Clusters
Heyot <i>et al</i> (1974)	Short range of particle correlations
Tsallis (1988)	Thermodynamic state of particles
Becattini <i>et al</i> (2002)	Transverse of particle momentum
Azmi (2015)	Charging state of particles

According to the quantum correlation which expresses a kind of contrast between classical and quantum mechanics, we provide an historical perspective of how the theory of correlations has connection with physics, which it is described by Biales *et al* (1972) as a research in the relation between two negative points in the process of



Heyot & Moral (1974) proved that the short range correlation between π^- 's in ISR energies are interpreted in terms of clustering the effects in the central rapidity region. Moral & Plaut (1974) showed that the average multiplicity and binomial moments of the clusters which decay distributions are proved to be increased with the

total multiplicity. Heyot *et al* (1974) in two different articles investigated on the short range part of two particle correlations which it results from the phase space available in cluster decay.

Tsallis (1988) presented a generalized standard equation for entropy of material particle states, by utilization of finite values for calculated variables. Becattini *et al* (2002) studied about the transverse of particle momentum in the high range of energy collisions. They indicated that in the hadronization process for producing of different particle species, their momentum spectra are closely related phenomenon governed by one parameter. Azmi (2015) studied about the momentum of the charged particles which measured by the ATLAS and CMS for p-p collisions by usage of Tsallis thermodynamics.

Research Methodology

Both of classical and quantum mechanics can be viewed as statistical theories respecting to measurements carried out in each way, but deterministic-nature added to quantum investigations is known as a privilege in superseded methods. That’s because of the emergence of the interference phenomenon for micro-objects, which is found in waves function at classical domains and exclusive concepts of particles is the realm of fuzzy where both of “particles” and “waves” are not mutually exclusive concepts. In the following by deriving the Schrodinger equation from the Hamilton-Jacobi equation, the fuzzy logic is applied to prediction of results.

For a classical particle in a conservative field the Hamilton-Jacobi equation (HJE) can be derived from Newton’s second law, in which the motion of a particle can be represented as a wave, for this reason as mentioned before the HJE is considered the closest approach of classical to quantum mechanics, and is a first order, non-linear partial differential equation

$$H + \frac{\partial S}{\partial t} = 0 \tag{1}$$

$$H = H(q_1, \dots, q_n; \frac{\partial S}{\partial q_1}, \dots, \frac{\partial S}{\partial q_n}; t) \tag{2}$$

$$S = S(q_1, \dots, q_n, t) \tag{3}$$

Where q_i are the N generalized coordinates ($i = 1, 2, \dots, N$) which define the configuration of the system, and t is time, S is as the generating function for a canonical transformation which based upon the derivatives of S the generalized momentum can be appeared. Remarkably, the function S is equal to the classical action statement and in wave functions and is expressed as the following

$$S = \int_{t_0}^{t_1} (\frac{p^2}{2m} - V) dt \tag{4}$$

Which the function is the well-known definition of the action for a particle moving in the potential field V, where p is the particle momentum of the material movement

with the mass (m) after lapse of a definite time. The remarkable points in particle movements are;

- i) They are expressed in imaginary forms, and
- ii) The iteration value (ω) is defined in approximation of movements per the following formula; $\omega = 2\pi\nu$, where ν is the frequency of dislocation.

By determination of ψ as the time-dependent wave function, the S can be expressed as; $S = K \ln \psi$ where K is the scaling factor of the wave and by utilization of the Schrodinger number h, the ψ function is as $\psi = \exp(\frac{iS}{\hbar})$, where $\hbar = 2\pi h$. Therefor having assumed to move of particle in a plane, the wave functions along each axis are represented by ψ and ψ^* , and by multiplying wave functions at coordinates of surface, the following formula is produced

$$\psi\psi^* = \frac{1}{2phb^2} \frac{1}{\sqrt{1+\frac{t}{T}+h^2}} \exp[\frac{-x+v_0t}{b^2(1+\frac{t}{T}+h^2)}] \tag{5}$$

Where T is the initial moment of time, t is any subsequent moment of time, b is the half width of frequent movement, $v_0 = \frac{x}{T}$ and x_0 is the initial state of particle. Basis of the functions definitions in a wave-like behavior is the non-linear movement with order 2 in quantum theories. Figure 1 is showing an example for approximation of a point laid on a wave in a surface with two coordinates of x and y, where $y = \frac{1}{x}$.

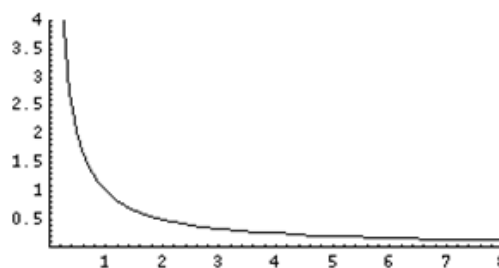


Figure 1 Wave path in 2D area

Based on Granik and Caulfield’s (1995) results, the relative degree of membership for any two points in space is 1. In other word, the result of $\psi\psi^*$ is constant and equal with 1.

$$\int_{-\infty}^{\infty} \psi\psi^* dv = \sum a_k a_k^* = 1 \tag{6}$$

Where $a_k a_k^*$ are projections onto the respective axes. Based on the last finding, by integrating from the related formula of $\psi\psi^*$, finally an equation for approximate calculation of particle momentum is resulted as

$$p = \frac{\sqrt{2+h^2} - \sqrt{1+\frac{t}{T}+h^2}}{2h} \tag{7}$$

And it is conducted that movement in frequency of 2T will be as

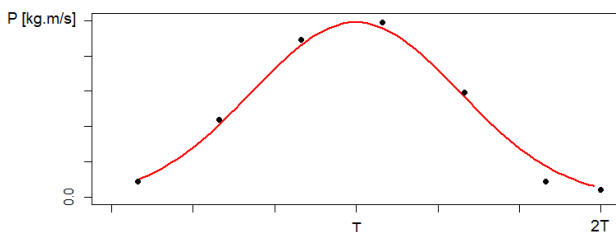


Figure 2 Particle Momentum vs initial acceleration time

In other words, since time from [0,T] particle will be forced to move from stationary point and reaches to maximum permissible speed at vicinity of neighboring particles and from time belonging to range of [T,2T], oscillations will be absorbed toward final stop at the time of 2T. Hence, as far as p is as a function of mass and speed of particle, so calculations are varied depending on atomic mass and type of surveyed substance. So by incorporation of fuzzification approach, it is possible to carry out the research in the relative area with dimensionless variables. Therefore, firstly the approximated speed of particle at the time of t (v_t) is assigned as follows and after that the relative speed (v_{rel}) at any time 1T to 2T is predicted by fuzzy assessment.

$$v_t = \frac{\sqrt{2+h^2} - \sqrt{1+\frac{t}{T}+h^2}}{2mh} \tag{8}$$

$$v_{rel} = \frac{v_{t \in [1T,2T]}}{v_{max}} = \frac{\sqrt{2+h^2} - \sqrt{1+\frac{t}{T}+h^2}}{\sqrt{2+h^2}} \tag{9}$$

As shown above, in the formula of v_{rel}, there is no effect from particle mass and the found result is not expressed in dimensions. In fuzzy logic, by usage of if-then rules it's provided to forecast the relative speed of particle during deceleration and the level of atomic surface friction is analyzed as well. In this manner for assigning a value in the last found formula for the parameter of $\frac{t}{T}$ from zero to 1, it's just required to introduce the initial and final points to fuzzy-logic controller and all of other points are approximated automatically. [Used platform is fuzzy logic toolbox in MATLAB software.]

$$\left\{ \begin{array}{l} \text{If } \frac{t}{T} = 0 \text{ then } v_{rel} \text{ is } \%100 \\ \text{If } \frac{t}{T} = 1 \text{ then } v_{rel} \text{ is } \%0 \end{array} \right.$$

The taken results with precision of 0.01 are shown in the following figure and for any respected value for $\frac{t}{T}$, it is possible to know the relative velocity of particle while energy losing.

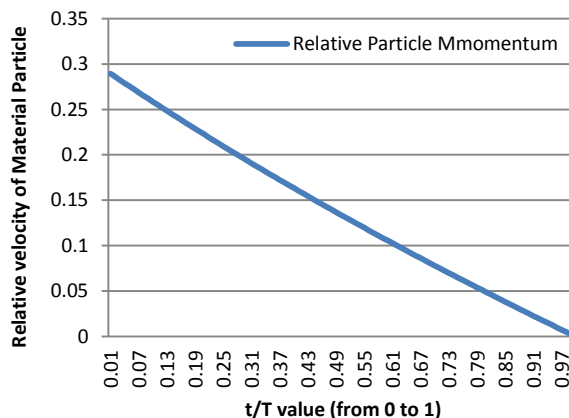


Figure 3 Tracking of particle velocity during deceleration

Table 2 Momentum values of particle at various time moments

CT = t/T	Relative Particle Momentum
0.01	0.28936648
0.02	0.285857157
0.03	0.282364995
0.04	0.278889745
0.05	0.275431163

As shown in the previous Figure and Table, because of concentration of particles in the molecular construction of substance, by applying any accelerative force over particles, immediately about %70 of received energy is absorbed by friction and just %30 of total energy is remained at inherent nature of particles for oscillations. It goes without saying that, the performed calculations is based on survey over pure materials and by adding impurities to structure as alloying elements, the conditions will be varied. But as future work, it is suggested to apply the different mass numbers of atoms to results and having them compared, because the remaining energies of particles for vibration is dumped sooner at large atoms and leads to more acceleration at atoms with smaller diameter.

Conclusion

Passage from classic to quantum mechanics in approximate determination of particles behavior in wave-like function is investigated in this paper. The research is carried based on fuzzy prediction if-then rules and derivation of Schrodinger equation in order 2, which produces the final equation of relative velocity in the research. According to the taken results for the relative speed of particle after initial acceleration, %70 of the received energy is dumped by molecular surface friction and depending on the size of particle, just around %30 of the remained energy is used for vibration until final recovery of stationary state.

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