International Journal of Multidisciplinary and Current Research ISSN: 2321-3124
Research Article Available at: http://ijmcr.com

Mathematical Modelling of Linear Complex Mechanical Systems by Inspection Method based on Force/Torque-Current Analogy

Abhilash V Nair[*] and Kannan Madhavan

[†]Department of Electrical & Electronics Engineering, APJ Abdul Kalam Kerala Technological University, LBS Institute of Technology for Women, Poojappura, Trivandrum-695012, Kerala, India

Received 14 Feb 2018, Accepted 15 April 2018, Available online 20 April 2018, **Vol.6 (March/April 2018 issue)**

Abstract

This paper introduces an easiest method for finding out the transfer function of complex mechanical systems with multi-input and multi-output. But at a time, one input and one output is significant. The method is an extension of inspection method of network solution by makes use of KVL. In the beginning step the available mechanical system has to be transformed into Laplace domain with associated differential co-efficient if any. Determine the Order of the system by considering the No. of masses/moment of inertia available in the system in turn determines the order of the transfer function matrix of the system. If once the transfer function matrix is formed, any combination of input and output can be selected to find out the transfer function by using Crammer's Rule. The method is a perfect fit one, easiest than state-space approach and fastest one for such systems states having trivial states.

Keywords: *Transfer Function, Multi-input Multi-output systems, KVL-Kirchhoff's Voltage Law, Laplace Transforms, State-Space, Crammer's Rule, Force/Torque-Current Analogy.*

1. Introduction

This method is an advancement of inspection method used for obtaining the mathematical model of electrical networks in circuit theory. There are two popular technics available for finding out the mathematical model of any electrical network, which are Mesh-Current analysis and Node-Voltage analysis. Among these two, the fore said method makes use of Mesh-Current Method.

A mechanical (translational) system is completely characterized by the system parameters like mass in kg, viscous friction coefficient in N-s/m, stiffness of the spring in N/m and there are equivalent circuits parameters of all three there in electrical system, which are resistance in ohm, inductance in henry and capacitance in farad. Among this group, the one-one equivalency depends on the type of analogy selected for the modeling. Similarly a mechanical rotational system can also be modeled with its parameters like moment of inertia, rotational viscous friction coefficient and rotational stiffness of the spring. This paper explains the transfer function modeling of complex systems by **force/torque-current** analogy.

2. Formation of System dynamics

Force-current analogy applies the principles of mesh-current analysis. In mesh-current analysis, apply KVL at each mesh to find out the linear mesh equations and represent in the form of matrix and the modeling is followed by the solution. In the beginning step obtain the system in transformed domain. If displacement(x) is the variable and is equivalent to charge in electrical circuit, the opposing force offers by

Mass (M) = $M\frac{d^2x}{dt^2}$ N.

Viscous friction coefficient (B) = $B\frac{dx}{dt}$N.

Spring stiffness (K) = Kx N. and corresponding transform equivalents are

Mass (M) = MS^2x N. (S-is the Laplace operator)

Viscous friction coefficient (B) = BSx N.

Spring stiffness (K) = Kx N.

In the mechanical systems, the mass can be treated as each mesh and all other elements can be treated as circuit elements around it. In conventional methods, it is very difficult and time consuming to calculate the transfer function of a higher order mechanical system. First step is to formulate the differential equation then rearrange it and convert it into Laplace domain. All these inter mediate steps can be eliminated and strait way write down the transfer function matrix from the transformed circuit.

*Corresponding author's ORCID ID: 0000-0000-0000-0000
DOI: https://doi.org/10.14741/ijmcr/v.6.2.11

3. Methodology Outline

a) Redraw the system with transformed parameters.
b) Identify the no of masses in the system and determine the order of transfer function matrix and the order of the model matrix.
c) Number the input variables and output variables.
d) Construct the transfer function matrix.
e) Co-relate necessary input and output variables and calculate corresponding transfer function by using crammers rule.

4. Illustration

Consider the mechanical system in Fig.1 as an example.

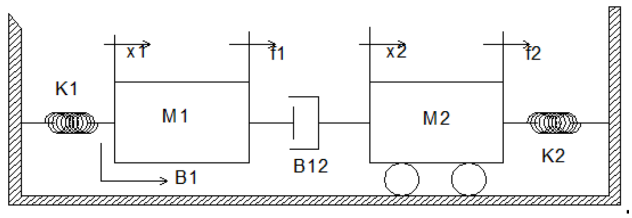

Fig.1 Mecanical System

In the example the time varying quantities are represented by small letters and constants are represented by capital letters. Two forces applied to two masses are f1 and f2, and corresponding displacements in each mass are x1 and x2. The system can be represented in transformed domain as follows.

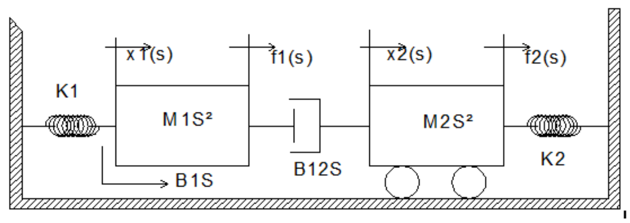

Fig. 2 Laplace Transformed Mechanical System

There are two masses in the system and each mass is multiplies with a second degree term. So the order of the overall system is four and the transfer function matrix has an order of two. Exact order will be determined by the values different parameters. Consider mass M1 as the first mesh, then the

T11 term of transfer function matrix (T)=$M1S^2$+(B1+B12)S+K1.

Since the friction in between two masses opposing the movement of each other so,

T12=-(B12S) =T21.

Consider M2 as the second mesh, T22=$M2S^2$+B12S+K2 and there are two inputs f1 and f2, two outputs x1 and x2.The transfer function matrix(T) can be formed as follows,

$$\begin{bmatrix} f1(s) \\ f2(s) \end{bmatrix} = \begin{bmatrix} M1S^2 + (B1+B12)S + K1 & -(B12S) \\ -(B12S) & M2S^2 + B2S + K2 \end{bmatrix} \begin{bmatrix} x1(s) \\ x2(s) \end{bmatrix}$$

From the above matrix any transfer function can be easily found out. For instance if f2=0, then two possible transfer functions are x1(s)/f1(s) and x2(s)/f1(s).Let us see how to calculate these two.

Solution X1(s)=Δ1/Δ(Crammers rule)

$$\Delta = \begin{vmatrix} M1S^2 + (B1+B12)S + K1 & -(B12S) \\ -(B12S) & M2S^2 + B2S + K2 \end{vmatrix}$$

$$\Delta 1 = \begin{vmatrix} f1(s) & -(B12S) \\ 0 & M2S^2 + B2S + K2 \end{vmatrix}$$

Therefore,

$$x1(s)/f1(s) = \frac{M2S^2+B2S+K2}{(M1S^2+(B1+B12)S+K1)(M2S^2+B2S+K2)-(B12S)^2}$$

Velocity (v)= $\frac{dx}{dt}$and v(s)=x(s).S and hence,

$$v1(s)/f1(s) = \frac{M2S+B2+K2/S}{(M1S+(B1+B12)+K1/S)(M2S+B2+K2/S)-(B12)}$$

Solution x2(s)=Δ2/Δ (Crammers rule)

$$\Delta = \begin{vmatrix} M1S^2 + (B1+B12)S + K1 & -(B12S) \\ -(B12S) & M2S^2 + B2S + K2 \end{vmatrix}$$

$$\Delta 2 = \begin{vmatrix} M1S^2 + (B1+B12)S + K1 & f1(s) \\ -(B12S) & 0 \end{vmatrix}$$

Therefore,

$$x2(s)/f1(s) = \frac{B12S}{(M1S^2+(B1+B12)S+K1)(M2S^2+B2S+K2)-(B12S)^2}$$

Velocity (v)= $\frac{dx}{dt}$and v(s)=x(s).S and hence,

$$v2(s)/f1(s) = \frac{B12}{(M1S+(B1+B12)+K1/S)(M2S+B2+K2/S)-(B12)^2}$$

Conclusion

Two transfer functions are obtained from a single model which will be same as the differential equation method gives. Once developed the mathematical model, formulation of transfer function repeatedly in between any variable is easy so that repeated manipulations of differential equations can be avoided. The method will correctly workout with rotational systems also. The method will be more beneficial while modeling very large systems. The transfer function matrix (T) obtained is a symmetrical matrix. So the method will fit perfectly for symmetrical systems. If a particular control system is symmetrical in nature then it must be a linear system. Further study over the nature of the transfer function matrix will leads to astonishing results regarding the system modeling method.

References

[1]. Guida, D.; Pappalardo, C.M. Forward and Inverse Dynamics of Nonholonomic Mechanical Systems Meccanica 2014, 49, 1547–1559.

[2]. Concilio, A.; De Simone, M.C.; Rivera, Z.B.; Guida, D. A new semi-active suspension system for racing vehicles. FME Trans. 2017, 45, 578–584.

[3]. Katsuhiko Ogata, Prentice Hall 2010, Modern Control Engineering, Instrumentation and controls series, Prentice-hall electrical engineering series, Instrumentation and controls series, Illustrated, ISBN-0136156738, 9780136156734

[4]. Smarajith Ghosh, PHI Learning Pvt. Ltd., 2005, Network theory: Analysis and Synthesis, ISBN-8120326385, 9788120326385

[5]. Shabana, A.A. Dynamics of Multibody Systems; Cambridge University Press: New York, NY, USA, 2013