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The Vibration of a Simply Supported Visco-Thermoelastic Nano-beam of Silicon Nitride Induced by Thermal Shock

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Abstract

In this article, the vibration of visco-thermoelastic isotropic homogeneous nano-beam has been studied. A general model of visco-thermoelasticity theory under simply supported conditions of one relaxation time has been used. The Laplace transformation has been applied for the governing equations. The inverse Laplace transformation has been considered by using Tzou procedure. When subjected to thermal shock loading and simply supported conditions, the numerical results have been validated for a visco-thermoelastic rectangular nano-beam of silicon nitride. Figures of this paper represent numerical results to describe the effects of the visco-thermoelastic parameters and the thickness of the nano-beam. The beam's thickness and the visco-thermoelastic parameters have been significant effects on all the studied state-functions.

Keyword: *Vibration; Visco-thermoelastic; Nano-beam; Silicon Nitride; Thermal shock; Fourier law*

Introduction

The concept of heat conduction has been considered using mathematical models for example dual-phase lag (DPL), which was suggested by Tzou [1,2]. The heat flux and temperature gradient have been recognized by DPL model. Several researchers used DPL model in heat transfer problems [3], physical systems [4-8]. One of the types of heat conduction is the theory of coupled thermoelasticity that consists of two differential partial equations: the equation of energy conservation and the equation of motion, based on Fourier's law of heat conduction [9-12]. Lord and Shulman introduced the relaxation time in case of an isotropic body by modifying Fourier's law of heat conduction, such that including the heat flux and its time derivative. In other word, non-Fourier's law of heat conduction replaces to Fourier's law [13]. In this case, the heat equation in the theory of coupled thermoelasticity is a hyperbolic equation that detects and removes the infinite speeds of propagation [14].

Mechanical signal processing, ultrasensitive mass detection, scanning probe microscopes, actuators, signal processing components and ultrafast sensors etc. are applications based on micro and nanoelectromechanical beam resonators [15-18]. Nano-beam's vibration is the most important of the micro/ nano beam resonator. Alghamdi [9] used the DPL thermoelasticity theory to studied the damping thermoelastic vibration of beam resonator with voids. Sharma and Grover [19] investigated the transverse thermoelastic vibration of isotropic and homogenous micro/nano thin beam resonators with voids. But the study of the damping thermoelastic vibration was done by Sun and Saka [20] for microplate circular resonators. They added factor $K = (1+\upsilon)/(1-2\upsilon)$ to formula of thermoelastic damping, so their formula becomes different from that of Lifshitz and Roukes [21], where υ is Poisson's ratio. Several researchers have investigated the heat transfer process and the vibration of nano-beams [22-26]. The study of the vibration of nano-beam gold subjected to thermal shock was done by Eman and Hamdi [23]. Kidawa [25] used the properties of the Green functions to study the effects of internal and external damping on beam's vibrations caused a moving heat source. Boley [24] investigated the affect a thermal shock on vibrations of a rectangular nano-beam that was simply supported. Manolis and Beskos [26] studied vibration of thermoelastic nano-beam's dynamic response under the effect of thermal loading by using a numerical method of analysis. Al-Huniti *et al*. [22] used a high-power moving laser beam to investigated the thermally induced displacements and stresses of heated rod and it's dynamical behaviour by using the Laplace transforms technique.

The study of visco-thermoelasticity has become important in mechanics. Biot [27,28] discussed the theory of visco-thermoelasticity and the principles in thermodynamics vibration. Drozdov [29] derived a thermo visco-elasticity model at finite strains. Ezzat and El-

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Karmany [30] used a model of thermo-viscoelasticity for isotropic material to studying the effects of volume properties of viscoelastic media in thermoelasticity. Carcione *et al*. [31] used a numerical algorithm in an elastic material using the Kelvin–Voigt model. Grover [32-34] studied vibrations in visco-thermoelastic micro-beam resonators. Grover and Seth [35] used a dual phase lagging model to study visco-thermoelastic microbeam resonators.

Basic Equations

We will consider a thermally isotropic homogenous conducting, Kelvin–Voigt type thermo-viscoelastic solid in systems of Cartesian coordinate initially unformed and at a uniform temperature T_0. The governing equations of motion and heat conduction in the context of generalized (non-Fourier) thermoelasticity for displacement vector $u(x,y,z,t)=(u,v,w)$ and temperature change $T(x,y,z,t)$, in the absence of heat sources and body forces, are given by [34]:

$$\sigma_{ij,j}=\rho\ddot{u}_i, i,j=x,y,z \tag{1}$$

$$\sigma_{ij}=\lambda\delta_{ij}e_{kk}+2\mu e_{ij}-\beta\delta_{ij}(T-T_0), i,j=x,y,z \tag{2}$$

$$KT_{,ii}=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho C_v T+\beta T_0\delta_{ij}e_{ij}\right), i,j=x,y,z \tag{3}$$

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right), i,j=x,y,z \tag{4}$$

$$\lambda=\lambda_0\left(1+\lambda_1\frac{\partial}{\partial t}\right), \mu=\mu_0\left(1+\mu_1\frac{\partial}{\partial t}\right), \beta=(3\lambda+2\mu)\alpha_T \tag{5}$$

Where ρ is the density, α_T is the coefficient of linear thermal expansion, λ_1,μ_1 are the viscoelastic relaxation times, τ_0 is the thermal relaxation time, λ_0,μ_0 Lamè's parameter in usual case, $C_v$is the specific heat and K is the thermal conductivity.

Problem Formulation

We will consider small flexural deflections of an elastic thin beam of length $\ell(0\le x\le \ell)$, width $b\left(-\frac{b}{2}\le y\le\frac{b}{2}\right)$ and thickness $h\left(-\frac{h}{2}\le z\le\frac{h}{2}\right)$, for which the x, y and z axes are defined along the longitudinal, width and thickness directions of the beam, respectively. In equilibrium, the beam is unstrained, unstressed, without a damping mechanism, and the temperature is T_0 everywhere [6].

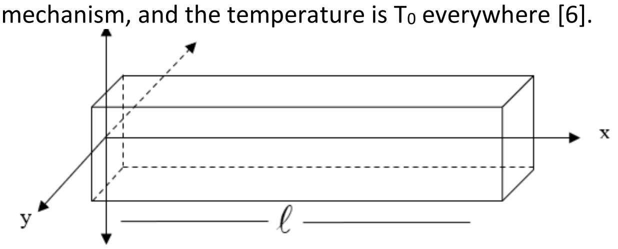

Fig. 1: Rectangular Nanobeam

In the present work, the Euler–Bernoulli assumption [32] is adopted, so, any plane cross-section, initially perpendicular to the axis of the beam remains plane and perpendicular to the neutral surface during bending. Thus, the displacements are given by

$$u=-z\frac{\partial w(x,t)}{\partial x}, v=0, w(x,y,z,t)=w(x,t) \tag{6}$$

The flexural moment of cross-section is given by

$$M(x,t)=(\lambda+2\mu)I\frac{\partial^2 w}{\partial x^2}+\beta M_T \tag{7}$$

and M_T is the thermal moment of the beam which is given by:

$$M_T=b\int_{-\frac{h}{2}}^{\frac{h}{2}}\theta z dz \tag{8}$$

$I=\frac{bh^3}{12}$is the moment of inertia of the cross-section about x-axis.

Hence, the thermally differential equation induced the beam lateral vibration expressed in the form [32]:

$$(\lambda+2\mu)I\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}+(3\lambda+2\mu)\alpha_T\frac{\partial^2 M_T}{\partial x^2}=0 \tag{9}$$

$w(x,t)$is the lateral deflection, $A=hb$is the cross-section area, and $\theta=(T-T_0)$ is the temperature increment of the resonator.

The non-Fourier heat conduction equation has the following form [32]:

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_v}{k}\theta+\frac{(3\lambda+2\mu)\alpha_T T_0}{k}e\right) \tag{10}$$

where $e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$is the volumetric strain which gives from (6) that:

$$e=-z\frac{\partial^2 w}{\partial x^2} \tag{11}$$

From the relation in (5), we have the following:

$$\lambda+2\mu=(\lambda_0+2\mu_0)\left(1+\beta_1\frac{\partial}{\partial t}\right), (3\lambda+2\mu)=(3\lambda_0+2\mu_0)\left(1+\beta_2\frac{\partial}{\partial t}\right) \tag{12}$$

where $\beta_1=\frac{(\lambda_0\lambda_1+2\mu_0\mu_1)}{(\lambda_0+2\mu_0)}$, $\beta_2=\frac{(3\lambda_0\lambda_1+2\mu_0\mu_1)}{(3\lambda_0+2\mu_0)}$ are the combination of the viscoelastic relaxation times parameters.

Because there is no heat flow across the lower and upper the beam surfaces, so that $\frac{\partial\theta}{\partial z}=0$ at $z=\pm h/2$. For a very thin beam and assuming the temperature varies in terms of a $sin(pz)$function along the thickness direction, where $p=\pi/h$, gives [36]:

$$\theta(x,z,t) = \vartheta(x,t)\, sin(pz) \quad (13)$$

Hence, equations (8), (9), and (13) gives:

$$(\lambda_0 + 2\mu_0)\left(1 + \beta_1 \frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + \frac{12\rho}{h^2}\frac{\partial^2 w}{\partial t^2} + \frac{12(3\lambda_0+2\mu_0)\alpha_T}{h^3}\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 \vartheta}{\partial x^2}\int_{-h/2}^{h/2} z\, sin(pz)\, dz = 0 \quad (14)$$

and equation (10) gives

$$\left(\frac{\partial^2 \vartheta}{\partial x^2} - p^2\vartheta\right) sin(pz) = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_v}{k}\vartheta\, sin(pz) - \frac{(3\lambda_0+2\mu_0)\alpha_T T_0}{k}\left(1 + \beta_2 \frac{\partial}{\partial t}\right) z \frac{\partial^2 w}{\partial x^2}\right) \quad (15)$$

After doing the integrations, equation (14) takes the form

$$(\lambda_0 + 2\mu_0)\left(1 + \beta_1 \frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + \frac{12\rho}{h^2}\frac{\partial^2 w}{\partial t^2} + \frac{24(3\lambda_0+2\mu_0)\alpha_T}{h\pi^2}\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 \vartheta}{\partial x^2} = 0 \quad (16)$$

In equation (15), we multiply the both sides by z and integrating with respect to z from $-\frac{h}{2}$ to $\frac{h}{2}$, then we obtain

$$\frac{\partial^2 \vartheta}{\partial x^2} - p^2\vartheta = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\varepsilon\vartheta - \frac{T_0 h\pi^2(3\lambda_0+2\mu_0)\alpha_T}{24k}\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2}\right) \quad (17)$$

where $\varepsilon = \frac{\rho C_v}{k}$.

Now, we will use the non-dimensional variables [19]:

$$\left(x', w', h', \ell'\right) = \varepsilon c_0(x, w, h, \ell), \left(t', \tau_0', \beta_1', \beta_2'\right) = \varepsilon c_0^2(t, \tau_0, \beta_1, \beta_2), \sigma' = \frac{\sigma}{\lambda_0 + 2\mu_0},$$
$$\vartheta' = \frac{\vartheta}{T_0}, c_0^2 = \frac{\lambda_0+2\mu_0}{\rho} \quad (18)$$

Then, we have

$$\left(1 + \beta_1 \frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4} + \varepsilon_1 \frac{\partial^2 w}{\partial t^2} + \varepsilon_2\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 \vartheta}{\partial x^2} = 0 \quad (19)$$

And

$$\frac{\partial^2 \vartheta}{\partial x^2} - \varepsilon_3\vartheta = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\left(\vartheta - \varepsilon_4\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2}\right) \quad (20)$$

$$\sigma_{xx} = \left(1 + \beta_1 \frac{\partial}{\partial t}\right) e - \gamma\left(1 + \beta_2 \frac{\partial}{\partial t}\right)\vartheta\, sin(pz) \quad (21)$$

where $\varepsilon_1 = \frac{12}{h^2}, \varepsilon_2 = \frac{24\gamma}{\pi^2 h}, \varepsilon_3 = p^2, \varepsilon_4 = \frac{\pi^2 h(3\lambda_0+2\mu_0)\alpha_T}{24k\varepsilon}, \gamma = \frac{(3\lambda_0+2\mu_0)\alpha_T T_0}{(\lambda_0+2\mu_0)}$.

(We have dropped the prime for convenience)

Formulation the Problem in the Laplace Transform Domain

The Laplace transform for equations (19) and (20), which is defined by the following formula will be applied:

$$\bar{f}(s) = \int_0^\infty f(t)e^{-st}\, dt \quad (22)$$

Hence, we obtain the following system of differential equations:

$$(1 + \beta_1 s)\frac{\partial^4 \bar{w}}{\partial x^4} + \varepsilon_1 s^2 \bar{w} + \varepsilon_2(1 + \beta_2 s)\frac{\partial^2 \bar{\vartheta}}{\partial x^2} = 0 \quad (23)$$

And

$$\frac{\partial^2 \bar{\vartheta}}{\partial x^2} - \varepsilon_3\bar{\vartheta} = (s + \tau_o s^2)\left(\bar{\vartheta} - \varepsilon_4(1 + \beta_2 s)\frac{\partial^2 \bar{w}}{\partial x^2}\right) \quad (24)$$

$$\bar{\sigma}_{xx} = (1 + \beta_1 s)\bar{e} - \gamma(1 + \beta_2 s)\bar{\vartheta}\, sin(pz) \quad (25)$$

$$\bar{e} = -z\frac{\partial^2 \bar{w}}{\partial x^2} \quad (26)$$

Among applying the Laplace transform, we used the following initial conditions:

$$\vartheta(x,0) = w(x,0) = \frac{\partial\vartheta(x,0)}{\partial t} = \frac{\partial w(x,0)}{\partial t} = 0 \quad (27)$$

We can re-write the above system to be in the forms:

$$(D^4 + \varepsilon_5)\bar{w} = -\varepsilon_6 D^2\bar{\vartheta} \quad (28)$$

And

$$(D^2 - \varepsilon_7)\bar{\vartheta} = -\varepsilon_8 D^2\bar{w} \quad (29)$$

where $D^r = \frac{\partial^r}{\partial x^r}, \varepsilon_5 = \frac{\varepsilon_1 s^2}{(1+\beta_1 s)}, \varepsilon_6 = \frac{\varepsilon_2(1+\beta_2 s)}{(1+\beta_1 s)}, \quad \varepsilon_7 = \varepsilon_3 + (s + \tau_o s^2), \varepsilon_8 = \varepsilon_4(s + \tau_o s^2)(1 + \beta_2 s)$

Eliminating $\bar{w}$ between the equations of the above system, then, we get

$$[D^6 - LD^4 + MD^2 - N]\bar{\vartheta} = 0 \quad (30)$$

In similar, eliminating $\bar{\vartheta}$ gives:

$$[D^6 - LD^4 + MD^2 - N]\bar{w} = 0 \quad (31)$$

where $L = \varepsilon_7 + \varepsilon_6\varepsilon_8, M = \varepsilon_5, N = \varepsilon_5\varepsilon_7$.

The solutions of the equations (30) and (31) take the forms:

$$\bar{\vartheta}(x,s) = -\varepsilon_8 \sum_{i=1}^{3} c_i k_i^2\, sinh\big(k_i(\ell - x)\big) \quad (32)$$

And

$$\bar{w}(x,s) = \sum_{i=1}^{3} c_i(k_i^2 - \varepsilon_7)\, sinh\big(k_i(\ell - x)\big) \quad (33)$$

where $\pm k_1, \pm k_2, \pm k_3$ are the roots of the characteristic equation.

$$k^6 - Lk^4 + Mk^2 - N = 0 \quad (34)$$

To calculate the constants $c_i, i = 1,2,3$, we must apply any set of boundary conditions, so we consider that the beam is thermally shocked and simply supported as following:

$$w(0,t) = \frac{\partial^2 w(0,t)}{\partial x^2} = 0, \vartheta(0,t) = \vartheta_0 H(t) \tag{35}$$

And

$$w(\ell,t) = \frac{\partial^2 w(\ell,t)}{\partial x^2} = \vartheta(\ell,t) = 0 \tag{36}$$

where H(t) is the unit step function and θ_0 is constant which gives the strength of the thermal shock.

Apply the Laplace transform, we have

$$\bar{w}(0,s) = \frac{\partial^2 \bar{w}(0,s)}{\partial x^2} = 0, \bar{\vartheta}(0,s) = \frac{\vartheta_0}{s} \tag{37}$$

and

$$\bar{w}(\ell,s) = \frac{\partial^2 \bar{w}(\ell,s)}{\partial x^2} = \bar{\vartheta}(\ell,s) = 0 \tag{38}$$

Then, we obtain the following system of linear equations:

$$\sum_{i=1}^{3} c_i k_i^2 \sinh(k_i \ell) = -\frac{\vartheta_0}{s\varepsilon_8} \tag{39}$$

$$\sum_{i=1}^{3} c_i (k_i^2 - \varepsilon_7) \sinh(k_i \ell) = 0 \tag{40}$$

And

$$\sum_{i=1}^{3} c_i (k_i^2 - \varepsilon_7) k_i^2 \sinh(k_i \ell) = 0 \tag{41}$$

After solving the above system equations, then, we get the solutions in the Laplace transform domain as:

$$\bar{\vartheta}(x,s) = \frac{\vartheta_0}{s\varepsilon_7}\begin{bmatrix} \frac{(\varepsilon_7-k_2^2)(\varepsilon_7-k_3^2)k_1^2}{(k_1^2-k_2^2)(k_1^2-k_3^2)\sinh(k_1\ell)}\sinh\big(k_1(\ell-x)\big) + \\ \frac{(\varepsilon_7-k_1^2)(\varepsilon_7-k_3^2)k_2^2}{(k_2^2-k_1^2)(k_2^2-k_3^2)\sinh(k_2\ell)}\sinh\big(k_2(\ell-x)\big) + \\ \frac{(\varepsilon_7-k_1^2)(\varepsilon_7-k_2^2)k_3^2}{(k_3^2-k_1^2)(k_3^2-k_2^2)\sinh(k_3\ell)}\sinh\big(k_3(\ell-x)\big) \end{bmatrix} \tag{42}$$

$$\bar{w}(x,s) = -\frac{\vartheta_0(\varepsilon_7-k_1^2)(\varepsilon_7-k_2^2)(\varepsilon_7-k_3^2)}{s\varepsilon_7\varepsilon_8}\begin{bmatrix} \frac{1}{(k_1^2-k_2^2)(k_1^2-k_3^2)\sinh(k_1\ell)}\sinh\big(k_1(\ell-x)\big) + \\ \frac{1}{(k_2^2-k_1^2)(k_2^2-k_3^2)\sinh(k_2\ell)}\sinh\big(k_2(\ell-x)\big) + \\ \frac{1}{(k_3^2-k_1^2)(k_3^2-k_2^2)\sinh(k_3\ell)}\sinh\big(k_3(\ell-x)\big) \end{bmatrix} \tag{43}$$

And

$$\bar{e}(x,s) = \frac{z\vartheta_0(\varepsilon_7-k_1^2)(\varepsilon_7-k_2^2)(\varepsilon_7-k_3^2)}{s\varepsilon_7\varepsilon_8}\begin{bmatrix} \frac{k_1^2}{(k_1^2-k_2^2)(k_1^2-k_3^2)\sinh(k_1\ell)}\sinh\big(k_1(\ell-x)\big) + \\ \frac{k_2^2}{(k_2^2-k_1^2)(k_2^2-k_3^2)\sinh(k_2\ell)}\sinh\big(k_2(\ell-x)\big) + \\ \frac{k_3^2}{(k_3^2-k_1^2)(k_3^2-k_2^2)\sinh(k_3\ell)}\sinh\big(k_3(\ell-x)\big) \end{bmatrix} 00 \tag{44}$$

The Stress and the Strain-Energy

The stress-strain energy which is generated on the beam is given by:

$$W(x,z,t) = \sum_{i,j=1}^{3} \frac{1}{2}\sigma_{ij}e_{ij} = \frac{1}{2}\sigma_{xx}(x,z,t)e(x,z,t) \tag{45}$$

Hence, we have:

$$W(x,z,t) = \frac{1}{2}\left[L^{-1}\left(\bar{\sigma}_{xx}(x,z,s)\right)\right]\left[L^{-1}\left(\bar{e}(x,z,s)\right)\right], \tag{46}$$

where $L^{-1}[\bullet]$is the inversion of Laplace transform.

Numerical Inversion of the Laplace Transform

We use the Riemann-sum approximation method to determine the solutions in the time domain and obtain numerical results. In this method, we can invert any function in the Laplace domain to the time domain as:

$$f(t) = \frac{e^{\kappa t}}{t}\left[\frac{1}{2}\bar{f}(\kappa) + Re\sum_{n=1}^{N}(-1)^n \bar{f}\left(\kappa + \frac{in\pi}{t}\right)\right] \tag{47}$$

where Re is the real part and $i$is an imaginary number unit. For faster convergence, many numerical experiments have shown that $\kappa t \approx 4.7$ Tzou [2].

Numerical Results and Discussion

we will discuss a numerical result. The physical constants of silicon nitride used as the thermoelastic material are set to the following values [34]:

$k = 43.5W/(mK)$, $\alpha_T = 2.71\ (10)^{-6}K^{-1}$, $\rho = 3200kg/m^3$, $T_0 = 293K$, $C_v = 630J/(kgK)$, $\lambda_0 = 217 \times 10^9 N/m^2$, $\mu_0 = 108 \times 10^9 N/m^2$, $\tau_0 = 4.32 \times 10^{-13}$S, $\lambda_1 = \mu_1 = 6.89 \times 10^{-13}s$.

We will assume that the aspect ratios of the beam as $\ell/h = 5$, $b = h/2$ and the range of the beam length is $\ell\ (1 - 100) \times 10^{-12}m$ for the nanoscale beam. The original time t and the relaxation time τ_0 of order 10^{-12}sec and 10^{-14}sec, respectively.

The figures were set by using the non-dimensional variables for nano-beam length $\ell = 1.0$, $\theta_0 = 1.0$ $z = h/4$and$t = 1.0$.

Figures 2-6 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, for the Biot model (the model based on Fourier law of thermoelasticity heat conduction) for thermoelastic case and visco-thermoelastic case. It has been noted that the temperature increment is nearly the same for the two cases, while the visco-thermoelastic parameters are significant effect on the lateral vibration, the deformation, the stress, and the stress-strain energy distributions. The peak points of the lateral vibration and the deformation decrease in the case of visco-thermoelasticity, while the absolute value of the stress increases in the same case. The peak point of the stress-strain energy distribution increases in the context of visco-thermoelastic model.

Figures 7-11 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, for the Lord-Shulman model (L-S) (the model based on non-Fourier law of thermoelasticity heat conduction) for the thermoelastic case and the visco-thermoelastic case. It has been noted that the visco-thermoelastic parameters are significant effect on temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions. The peak points of the temperature increment, the lateral vibration and the deformation decrease in the case of visco-thermoelasticity, while the absolute value of the stress increases in the same case. The peak point of the stress-strain energy distribution increases in the context of visco-thermoelastic model.

Figures 12-16 represent the temperature increment, the lateral vibration, the deformation, the stress, and the stress-strain energy distributions, respectively, for the Lord-Shulman model (L-S) for the visco-thermoelastic case with different values of the beam's thickness $z = (h/4, h/6)$ to stand on the thickness effect on all the studied functions. It has been noted that the visco-thermoelastic parameters are significant effect on temperature increment, the deformation, the stress, and the stress-strain energy distributions while its effect on the lateral vibration is null. The peak points of the temperature increment, the deformation, the stress, and the stress-strain energy increase in the case of $z = h/4$, while it is almost the same for the lateral vibration.

Conclusion

When the visco-thermoelastic beam has been thermally shocked and simply supported, the visco-thermoelastic parameters have significant effects on the lateral vibration, the deformation, the stress, and the stress-strain energy distributions and null effect on the temperature increment distribution in the context of Fourier law of heat conduction. In the context of the non-Fourier law of heat conduction, the visco-thermoelastic parameters and the thickness of the nano-beam have important effects on all the studied state-functions.

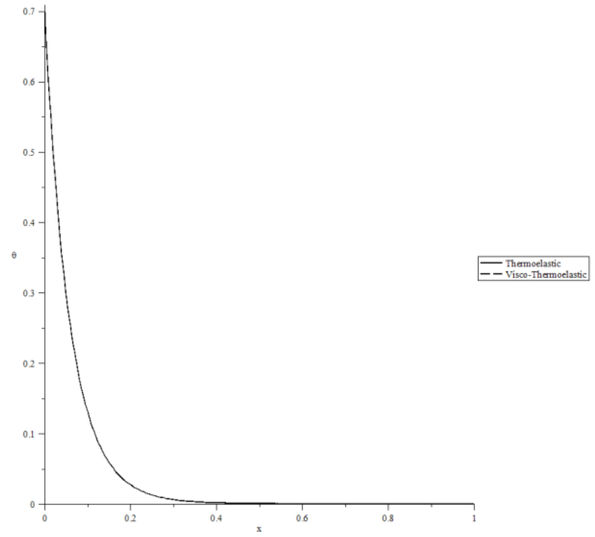

Fig. 2: The temperature increments distribution for Biot model

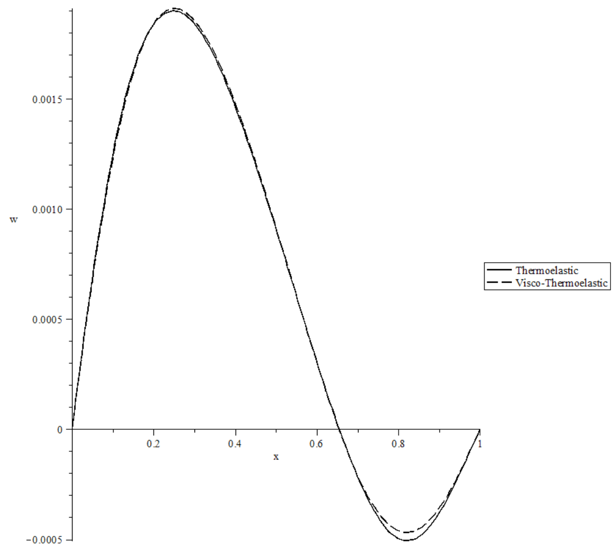

Fig. 3: The lateral deflection distribution for Biot model

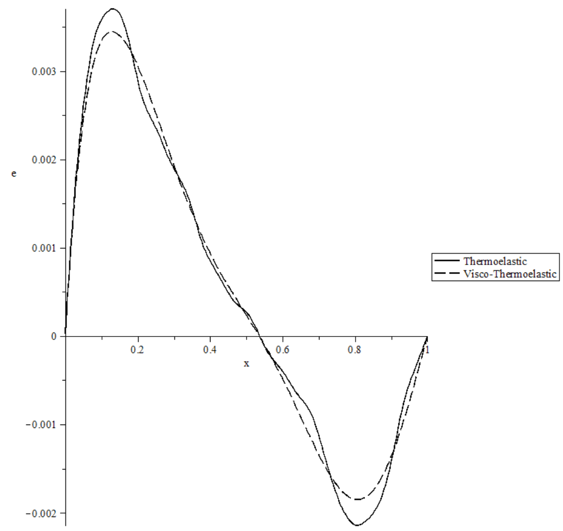

Fig. 4: The deformation distribution for Biot model

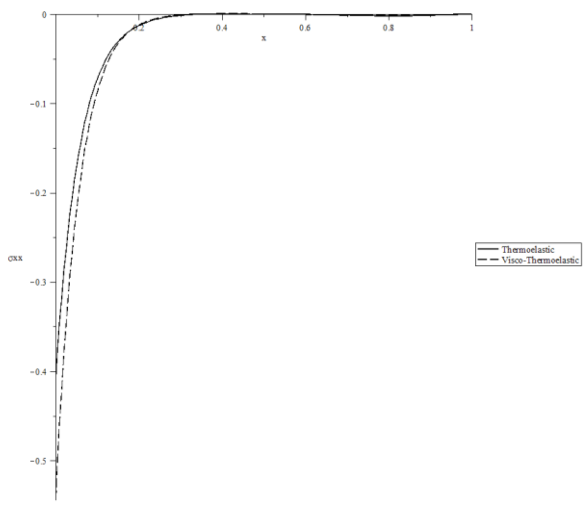

Fig. 5: The stress distribution for Biot model

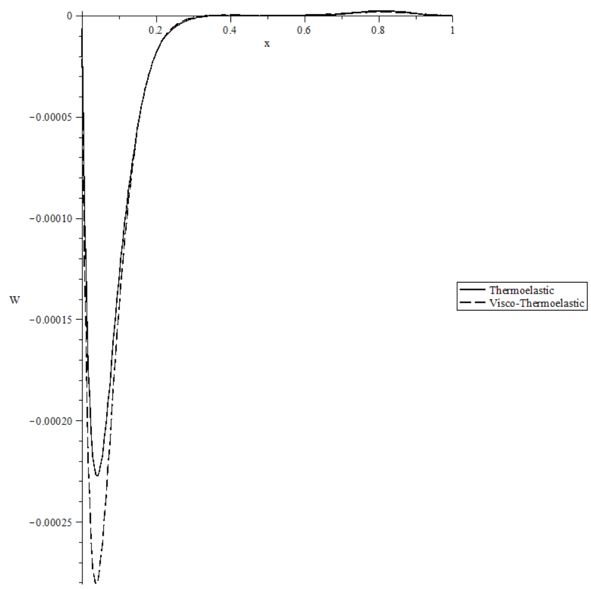

Fig. 6: The stress-strain energy distribution for Biot model

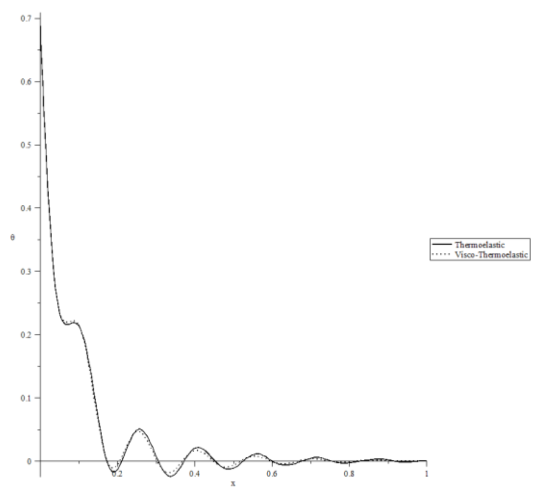

Fig. 7: The temperature increments distribution for L-S model

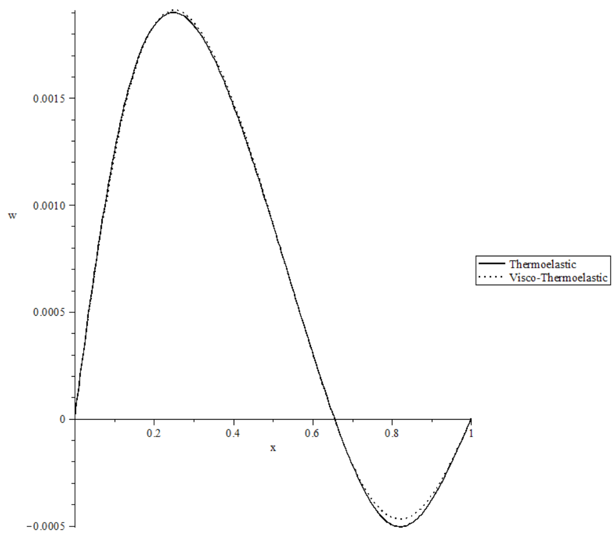

Fig. 8: The lateral deflection distribution for L-S model

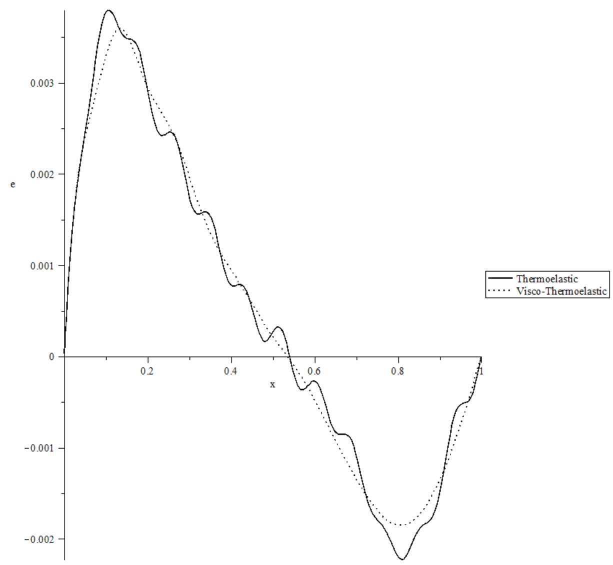

Fig. 9: The deformation distribution for L-S model

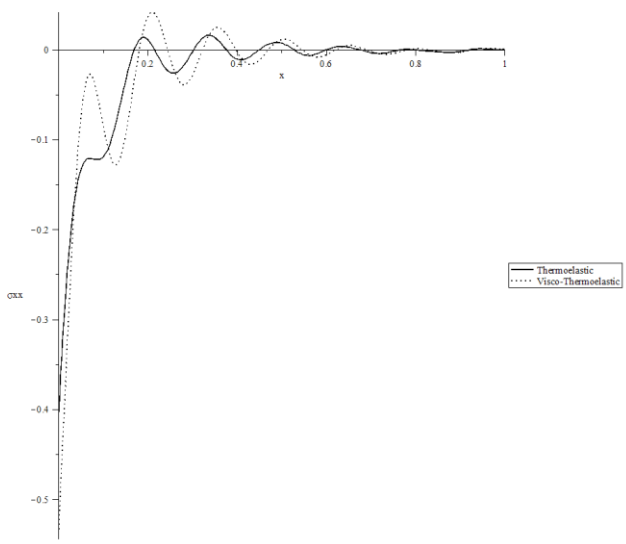

Fig. 10: The stress distribution for L-S model

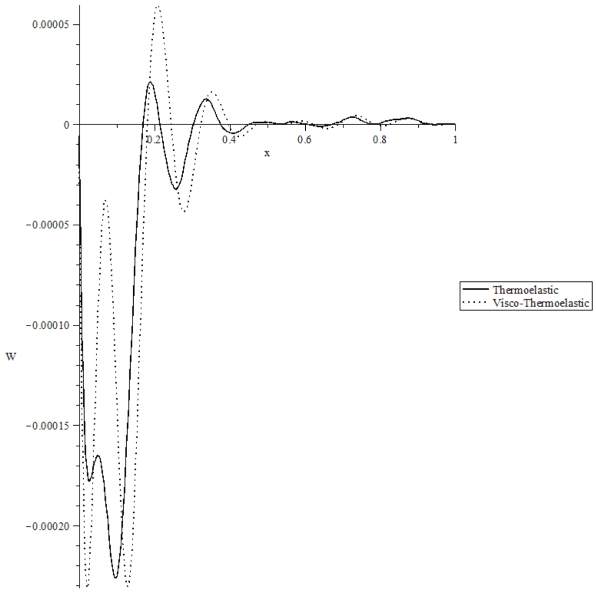

Fig. 11: The stress-strain energy distribution for L-S model

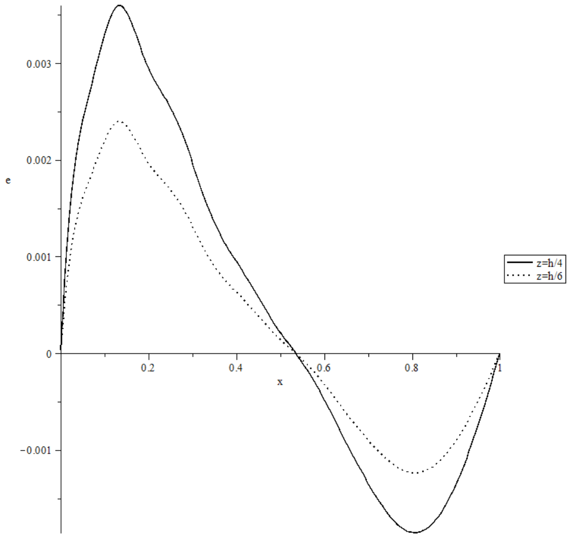

Fig. 14: The deformation distribution with variance thickness h

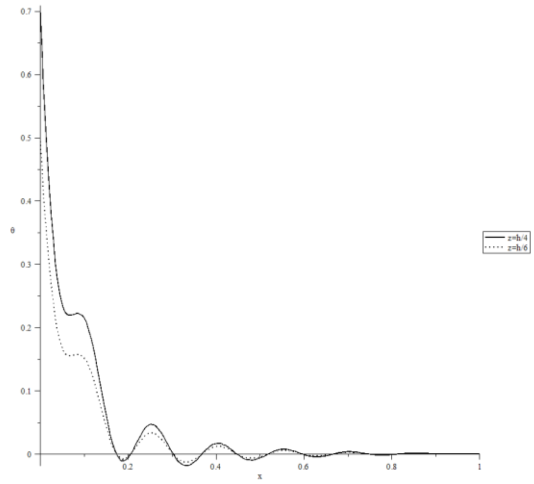

Fig. 12: The temperature increments distribution with variance thickness h

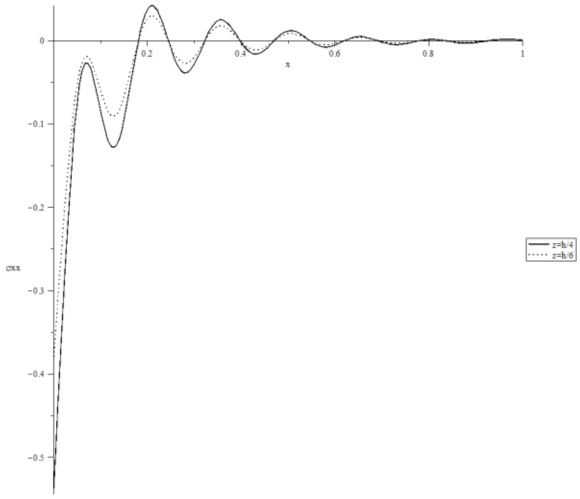

Fig. 15: The stress distribution with variance thickness h

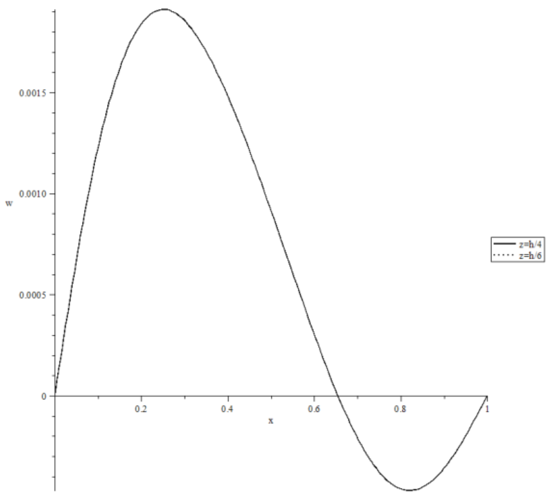

Fig. 13: The lateral deflection distribution with variance thickness h

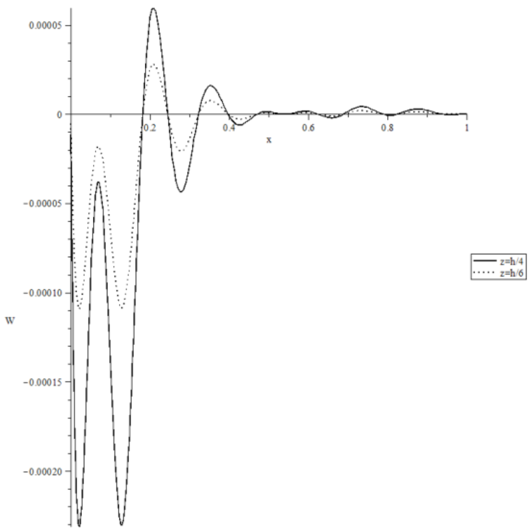

Fig. 16: The stress-strain energy distribution with variance thickness h

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