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Design of Low pass FIR Filters using Kaiser Window Function with variable parameter Beta (β)

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Abstract

The paper deals with designing of low pass Finite Impulse Response filter using Kaiser Window with variable parameter beta (β) between 0 and 4, further a comparative study is made at different filter orders so as to control the transition bandwidth. The Kaiser window is chosen as it contains an adjustable parameter with which main lobe width and correspondingly minimum stop band attenuation of designed filter can be controlled.

Keywords: *FIR filter, Kaiser window, Fourier Transform*

1. Introduction

A Digital filter finds applications in digital signal processing and can be implemented using either infinite impulse response (IIR) or finite impulse response (FIR) methods. The designing of FIR filter is chosen as they have several advantages [1].The designing includes truncating the infinite duration impulse response using a set of time limited weighted window function. But this technique may result in poor convergence of the truncated series, particularly in the vicinity of discontinuities, making the method unsatisfactory for approximating digital filters. This effect of abrupt truncation is known as the Gibbs's phenomenon [1], i.e., there is some percentages of overshoot, undershoot, and ripple before and after an approximated discontinuity.

Therefore, to remove the presence of large oscillations in both passband and stopband we have chosen Kaiser window function that contains a taper and decays towards zero gradually.

2. Kaiser Window

The width of the main lobe is inversely proportional to the length of the filter. The attenuation in the side lobe is, however, independent of the length and is function of the type of the window. A complete review of many window functions and their properties was presented by Harris [2].

Therefore the length of the filter must be increased considerably to reduce the main lobe width and to achieve the desired transition band.

Kaiser has chosen a class of windows having properties closely approximating those of the prolate spheroidal wave functions. This family of windows, known as the Kaiser windows is defined by

$$w_k(\beta,n)=\begin{cases}\dfrac{I_0\left\{\beta\left[1-\left(\dfrac{2n}{N-1}\right)^2\right]^{1/2}\right\}}{I_0(\beta)} & -\dfrac{N-1}{2}\le|n|\le\dfrac{N-1}{2}\\ 0 & otherwise\end{cases} \tag{1}$$

Where N is window length and $I_0(x)$ is the modified Bessel function of the first kind of order zero, given by:

$$I_0(x)=\sum_{k=0}^{x}\left[\frac{(x/2)^k}{k!}\right]^2 \tag{2}$$

The Kaiser window provides the designer considerable flexibility in meeting the filter specifications.

3. Low pass FIR filter design

This section deals with designing of Low pass FIR filter with higher order using Kaiser window with variable beta.

3.1 Low Pass Filters

The impulse response h(n) of a Lowpass filter is given by

$$h(n)=\begin{cases}\dfrac{2f_c}{F} & for\quad n=0\\ \dfrac{2f_c}{F}\times\dfrac{\sin\left(\dfrac{2\pi nf_c}{F}\right)}{\dfrac{2\pi nf_c}{F}} & for\quad n\neq 0\end{cases} \tag{3}$$

where f_c is the ideal cut off frequency, F is the sampling frequency and n are the number of samples.

3.2 Design specification of low pass FIR

Cutoff frequency=20Hz
Sampling frequency=100Hz
Filter order=38 and 48

4. Results

The Kaiser window with beta 0 to 4, in time and frequency domain is plotted in figure 1-10. From the Table 1, it is clear that , with widow length 38, as the value of beta is increased the width of main lobes increases from 0.046875 at β=0 to 0.0625 at β=4 and side lobe attenuation decreases from -14.7dB at β=0 to -31dB at β=4 .While for window length 48, the width of main lobes increases from 0.035156 at β=0 to 0.046875 at β=4 and side lobe attenuation decreases from -18.7dB at β=0 to -30.8dB at β=4.

When the same window function is applied to lowpass FIR filter of order 38 and 48, it gives response as shown in figures11-20. From Table 2, it is clear that at filter order 38, with increase in beta transition width of filter is increased from 0.0477295 at β=0 to 0.095703 at β=4 while stopband attenuation decreases .While for filter order 48 , with increase in beta transition width of filter is increased from 0.0352254 at β=0 to 0.0723877 at β=4 while stopband attenuation decreases.

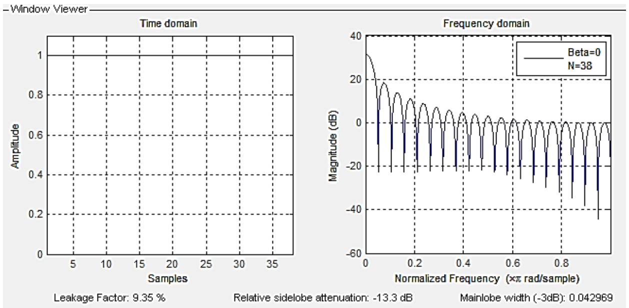

Fig 1: Magnitude Response of Kaiser window function at β=0 and N=38

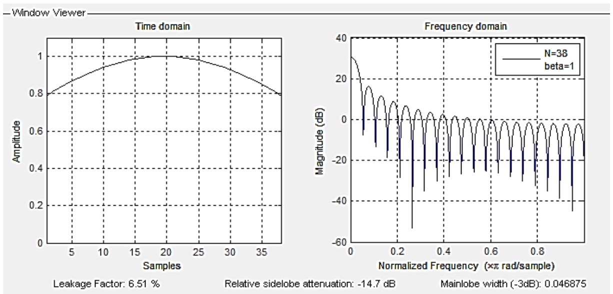

Fig 2: Magnitude Response of Kaiser window function at β=1 and N=38

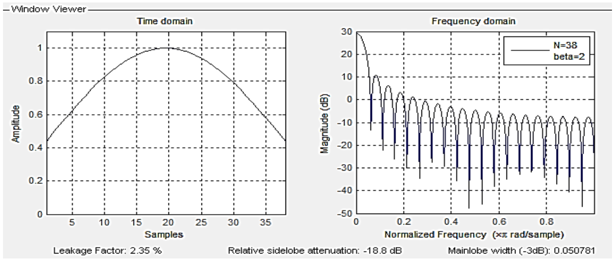

Fig 3: Magnitude Response of Kaiser window function at β=2 and N=38

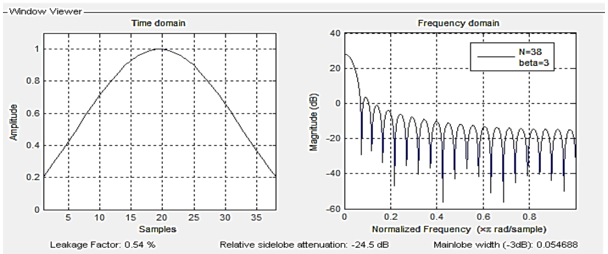

Fig 4: Magnitude Response of Kaiser window function at β=3 and N=38

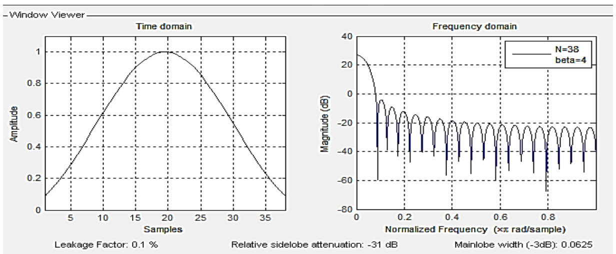

Fig 5: Magnitude Response of Kaiser window function at β=4 and N=38

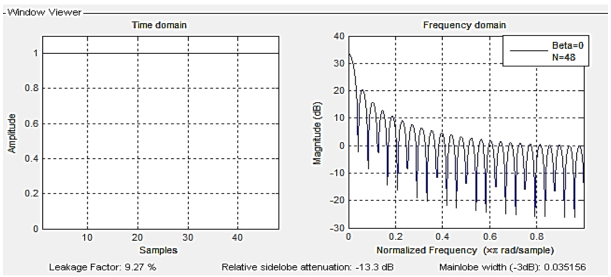

Fig 6: Magnitude Response of Kaiser window function at β=0 and N=48

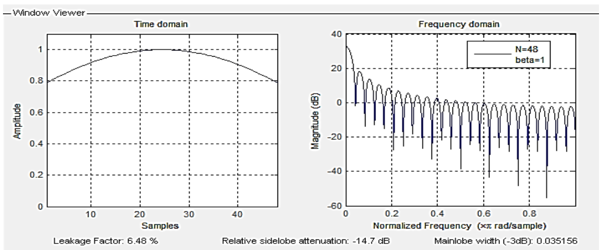

Fig 7: Magnitude Response of Kaiser window function at β=1 and N=48

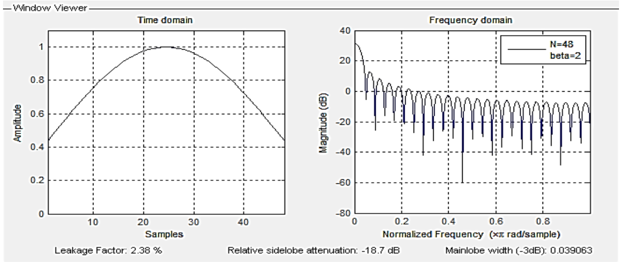

Fig 8: Magnitude Response of Kaiser window function at β=2 and N=48

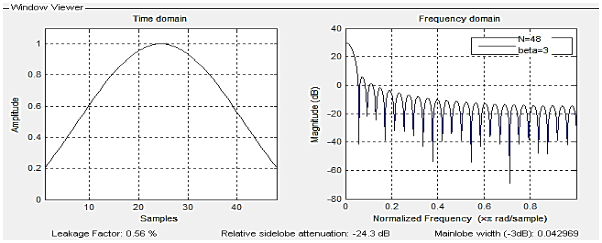

Fig 9: Magnitude Response of Kaiser window function at β=3 and N=48

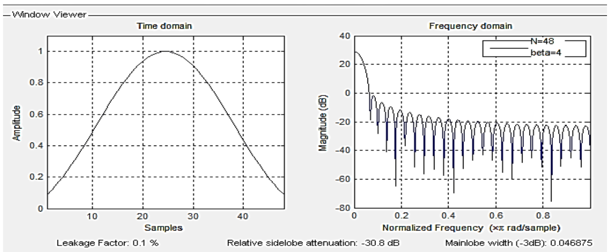

Fig 10: Magnitude Response of Kaiser window function at β=4 and N=48

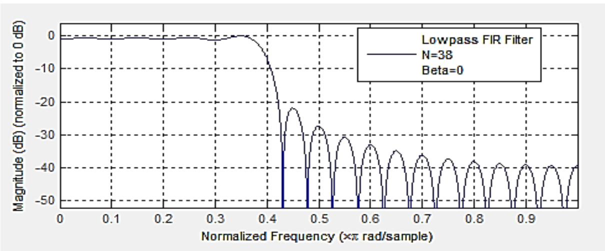

Fig 11: Magnitude Response of Lowpass FIR Filter at β=0 and N=38

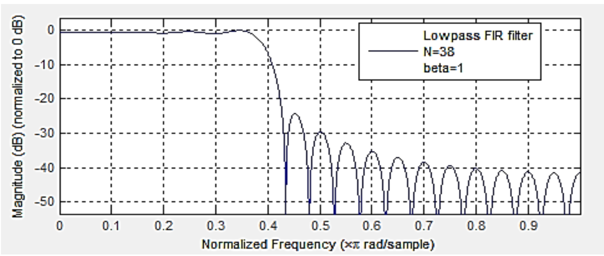

Fig 12: Magnitude Response of Lowpass FIR Filter at β=1 and N=38

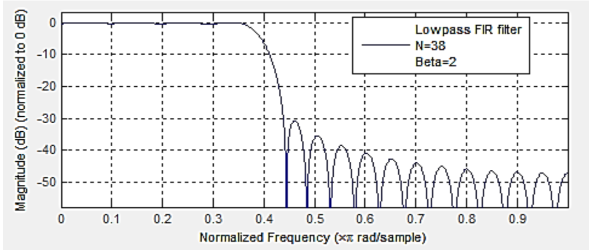

Fig 13: Magnitude Response of Lowpass FIR Filter at β=2 and N=38

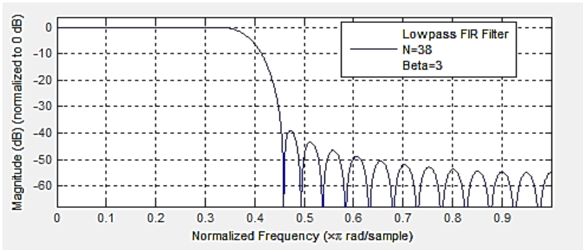

Fig 14: Magnitude Response of Lowpass FIR Filter at β31 and N=38

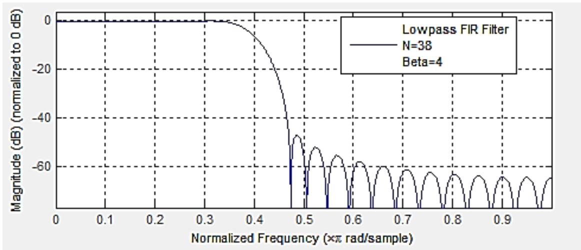

Fig 15: Magnitude Response of Lowpass FIR Filter at β=4 and N=38

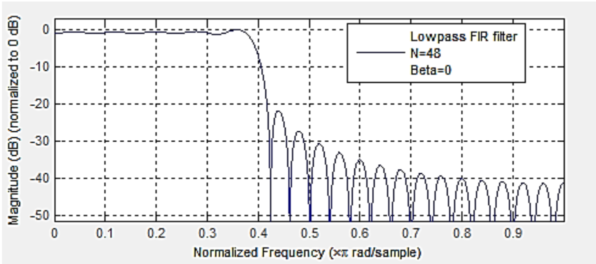

Fig 16: Magnitude Response of Lowpass FIR Filter at β=0 and N=48

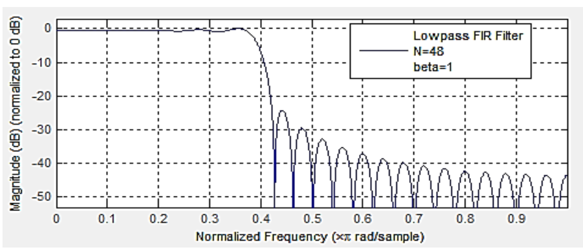

Fig 17: Magnitude Response of Lowpass FIR Filter at β=1 and N=48

Table 1: Comparison of Kaiser window function at different β & N

Variable parameter beta	length of window function	Time domain		Frequency domain	
		Max. Amp.	Min. Amp.	Main lobe width	Relative sidelobe attenuation dB
0	38	1	1	0.042969	-13.3
	48			0.035156	-13.3
1	38	1	.o8	0.046875	-14.7
	48			0.035156	-18.7
2	38	1	.04	0.050781	-18.8
	48			.0390630	-18.7
3	38	1	.02	0.054688	-24.5
	48			.0429690	-24.3
4	38	1	.01	0.625000	-31.0
	48			0.046875	-30.8

Table 2: Comparison of low pass FIR Filter using Kaiser window function at different β & N

Variable parameter (β)	Order of filter	Transition bandwidth	Sidelobe attenuation (dB)		No. of side lobes
			max	min	
0	38	0.047729	-21.95	-39.52	12
	48	0.035225	-21.96	-41.81	15
1	38	0.048096	-24.39	-41.85	12
	48	0.040893	-24.35	-43.51	15
2	38	0.062891	-30.81	-47.25	12
	48	0.050415	-30.44	-49.90	15
3	38	0.074340	-39.20	-55.05	12
	48	0.061645	-38.11	-56.43	15
4	38	0.095703	-47.22	-64.70	12
	48	0.072387	-46.79	-65.84	15

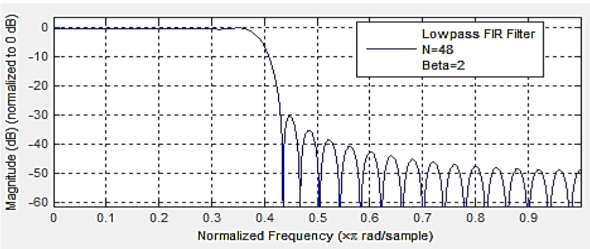

Fig 18: Magnitude Response of Lowpass FIR Filter at β=2 and N=48

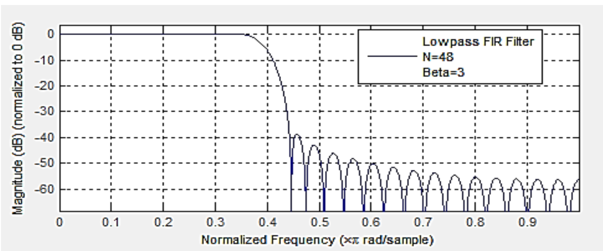

Fig 19: Magnitude Response of Lowpass FIR Filter at β=3 and N=48

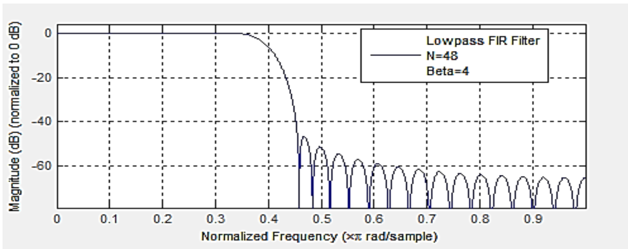

Fig 20: Magnitude Response of Lowpass FIR Filter at β=4 and N=48

Conclusion

The paper shows a comparative study of Kaiser window with variable parameter β & at different N. Further a lowpass FIR filter is designed using same window with same parameters .The study shows that as β is increased the main lobe width of Kaiser window increases and sidelobe attenuation decreases, at the same time minimum window length gives better result. But when same window in applied to a lowpass FIR filter, higher order filter gives better result as transition width decreases with increase in filter order. Moreover sidelobe attenuation decreases as filter order and β increases.

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