

## Mathematical Modelling of Thin Layer Solar Drying of Fish (*Bagrus bayad*)

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### Abstract

Solar drying experiments in thin layer of fish were conducted in Shambat, Sudan. A natural convective solar dryer consisting of solar collector (air heater) with air duct and drying chamber was used in the experiments. Resulted heated air was passed through a thin layer of fish. The changes in the mass of fish and principal drying parameters were recorded continuously every one hour interval from morning to evening in each test day. Drying curves obtained from experimental data were fitted to a number of mathematical models and the effect of drying air temperature and relative humidity on model constants and coefficients were evaluated by the regression analysis and compared to previously given models. The Lewis drying model was found to be satisfactorily describing solar drying curves of fish with a correlation coefficient ( $R^2$ ) of 0.9827

**Keywords:** Solar Dryer, Fish, Mathematical Models, Drying Curves.

### Introduction

Fish provides a good source of high quality protein and contains many vitamins and minerals; which are almost cheaper than any others type of meat and therefore available to large number of people (Fellows and Hamptom, 1992). Every year, large quantities of fish have been lost because of inadequate preservation and storage of meat. This is due to the high moisture content present in meat and consequently the rate of growth and development of micro-organisms; insects and mites increase very rapidly. Fish is an extremely perishable food, and both quantity and quality will be reduced after catching the fish. For example most fish become inedible within 12 hours at tropical temperatures; spoilage begins as soon as the fish dies (Fellows and Hamptom, 1992). Large scale production of food limits the use of open-air natural sun drying. Among these are lacks of ability to control the drying properly, weather uncertainties, high labour cost, large area requirements, and insect infestation, mixing with dust and others materials and so on. The solutions in solar energy collection devices or solar dryers have been proposed to utilize free renewable and non-polluting energy sources provided by the sun. The introduction of solar dryers in developing countries can reduce food losses and improve the quality of the dried product significantly compared to traditional drying methods (Muhlbauer, 1986; Togrul and

Pehlivan, 2002). The drying constant is the combination of the transport properties (thermal conductivity, thermal diffusivity, moisture diffusivity and interface heat and mass transfer coefficients) (Sokhansanj, 1984; Vegenas and Karathanos, 1993; Belessiotis, and Karathanos, 1999). The drying constant combines all the transport properties and may be defined by thin layer equations.

$$dM/dt = -k(M - M_e) \quad (1)$$

where M is the material moisture content, dry basis in kg water /kg dry solid at time t;  $M_e$  is the material moisture in equilibrium with drying air (no further evaporation of water); and t is the time in seconds; Belessiotis, and Karathanos, 1999).

Drying characteristics of the particular materials being dried and simulation models are needed in the design, construction and operation of drying system. Several researchers have developed simulation models for natural and forced convection solar drying systems (Diamante and Munro, 1993; Dincer, 1996; Exell, 1980; Tiris Ozbalta, Tiris and Dincer, 1994; Zaman and Bala, 1989 and Togrul and Pehlivan, 2002). Recently, there have been many studies on the drying behavior of various products such as grape (Yaldiz, Ertekin, and Uzan, 2001), potatoes (Diamante and Munro, 1993), onion (Sarsavadia, Sawhney, Pangavhane and Singh, 1999), green pepper, green bean and squash (Yaldiz, Ertekin, 2001) and rice

(Basunia and Abe, 2001). Thin layer equations describe the drying phenomena in a unified way, regardless of the controlling mechanisms. They have been used to estimate drying time of several products and to generalize drying curves (Midilli, Kucuk, and Yaper, 2002); (Togrul and Pehlivan, 2004). Therefore, the objective of this study to develop a mathematical model for the thin layer solar drying of the fish using an indirect natural convection solar dryer under ecological conditions typical of Shambat, Sudan.

**Materials and methods**

**Solar dryer**

Natural convective solar collector with the drying chamber were used in all the drying experiments Fig1 Dryer consisting of a solar collector (air heater) in rectangular shape and air duct to connect solar.



**Fig. 1** Natural convective solar dryer

The rectangular solar collector with the dimension of 180cm x90cm was constructed. It consists of an absorber plate made of metal sheets and painted in black to absorb maximum incident radiation. A water-white glass type was used as a transparent cover because it is a good transmitter for solar radiation; the glass also acts as a barrier to prevent wind and any others foreign materials to enter the solar dryer. The front opening of the collector was covered with a wire mesh to prevent the entry of any foreign materials to enter the solar collector. The collector was oriented to the south and tilted to form angle of 15.5° with the ground surface which is the angle of the latitude of the experimental site (Fig 1). Drying chamber is an enclosure in which product can be placed for drying. It is a box with three mesh trays. The dimensions of drying chamber were 100cm in width and 100cm in height. The top of the drying chamber had a pyramid shape with the central opening of 20cmx20cm for exiting the exhausted air after passing through fish. On the north side of the drying chamber there is a door used for loading or unloading the trays inside and outside the drying chamber. The inner side and the bottom of the

drying chamber were covered with Masonite of 3mm in thickness to reduce heat losses from the drying chamber to the surroundings. The outer surfaces of the drying chamber were also painted in black to absorb heat from the outer side and this will increase the temperature of the drying air inside the drying chamber.

**Solar drying experiments**

The solar drying experiments were carried out during the period of June to August 2010 in Shambat, Sudan. Each test started at 8:30 and continues till 17:30. Temperature and relative humidities of ambient air, heated air and exhausted air were measured and recorded. Mass of representative sample of fish was measured at hourly intervals during the experiments. Temperature measurements and recordings were made by thermocouples connected to Data logger enabling recordings in ±0.1c° accuracy. A representative sample was taken and warped with aluminum foil for determining initial moisture content of fish using an oven method (AOAC, 1984). Initial and final moisture content of fish were determined by an oven at 105c°. The moisture contents were reported as % wet basis or dry basis, which is more suitable in computation.

**Mathematical modelling of solar drying curves**

The obtained drying curves were processed to find the most convenient one among 5 different expressions defining drying rates given in table 1 by several authors. The moisture ratio however, was simplified to M/Mo instead of the (M-Me)/ (Mo- Me) because relative humidity of the drying air fluctuated continuously in solar drying (Diamante and Munro, 1991).

**Table 1** Mathematical models given by various authors for the drying curves

Model no.	Model equation.	Name	References
1	$MR = \exp(-kt)$	Lewis	Liu and Bakker-Arkema (1997) and O'Callaghan, Menzies and Bailey (1971)
2	$MR = \exp(-ktn)$	Page	Agrawal and Singh (1977) and Zhang and Litchfield (1991)
3	$MR = a \exp(-kt)$	Henderson and Babis	Chhninman (1984) and Westerman, White, and Ross (1973)
4	$MR = a \exp(-kt) + c$	Logarithmic	Yagctoglu, Degir mencioglu, and Cagaty (1999)
5	$MR = a \exp(-kt) + (1-a) \exp(-kbt)$	Diffusion	Kassem (1998)

**Statistical analysis validation**

A statistical tool such as linear regression was performed. The coefficient of determination (R<sup>2</sup>) was one of the primary criteria for selecting the best model to describe thin-layer drying curves of fish flakes. Also, there are some statistical parameters were done such as modeling

efficiency (EF), reduced chi-square ( $\chi^2$ ) and root mean square error (RMSE) to evaluate the goodness of the fit of the selected models. The lower the  $\chi^2$  and RMSE values and the higher (R2) values and EF were chosen as the criteria for goodness of the fit. These parameters were described in the following equations (Togrul and Pehlivan, 2002; Demir, et al., 2004).

$$\chi^2 = \frac{\sum_{i=1}^N (MR_{exp,i} - MR_{pre,i})^2}{N - n} \tag{2}$$

$$EF = \frac{\left( \sum_{i=1}^N (MR_{exp,i} - MR_{exp,mean})^2 \right) - \left( \sum_{j=1}^N (MR_{pre,j} - MR_{exp,in})^2 \right)}{\left( \sum_{i=1}^N (MR_{exp,i} - MR_{exp,mean})^2 \right)} \tag{3}$$

$$RMSE = \left( \frac{1}{N} \sum_{i=1}^N (MR_{pre,i} - MR_{exp,i})^2 \right)^{\frac{1}{2}} \tag{3}$$

Where:

MR<sub>exp,i</sub> = experimental moisture ratio at observation i, dimensionless

MR<sub>pre,i</sub> = predicted moisture ratio at this observation i, dimensionless

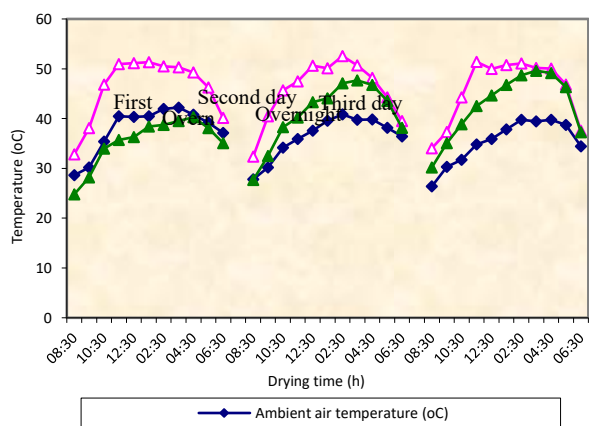
MR<sub>exp,mean</sub> = the mean experimental moisture ratio

N = number of observations

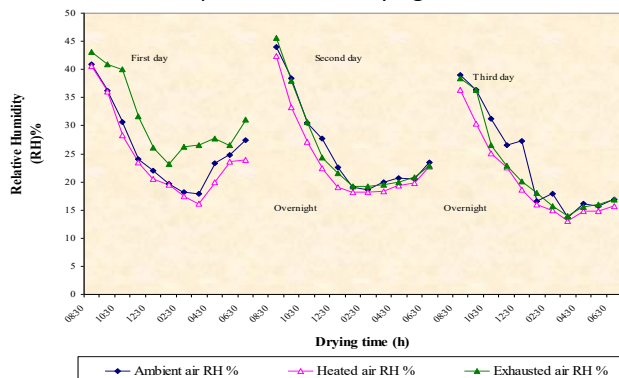
n = number of constants in the drying model

The effect of initial and final moisture content, drying air temperature and relative humidity of the air on the drying constants have been investigated in many studies (Agrawal and Singh, 1977; Henderson, 1974; Ozdemir and Devres, 1999; Pangavhane et al. 1999; Yaldiz and Erekin, 2001; Yaldiz et al., 2001 Zhang and Litchfield, 1991). In this study, the constant and coefficients of the best fitting model involving the drying variables such as temperatures and relative humidity of the drying air were determined. The effects of these variable on the constant and coefficients of drying expression were also investigated by linear regression analysis.

**Results and discussion**



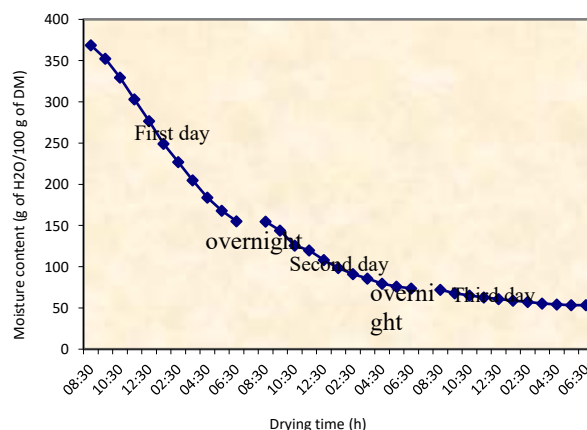
**Fig 2** Average ambient, heated and exhausted air temperatures with drying time



**Fig 3** Average ambient, heated and exhausted air relative humidities with drying time

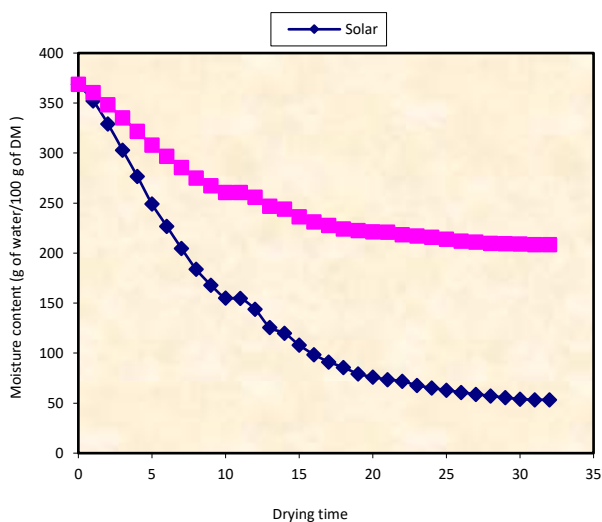
The weather conditions in the drying period are presented. Fig. 2 and 3 were shown the average ambient, heated and exhausted air temperatures and relative humidities respectively. The interruption of the lines in these figures represent the night period of the drying operation. The high temperature in solar dryer was obtained at noon due to the increase in solar radiation. The average solar radiation was about 20 MJ/m2/day while the average heated air temperature was about 46 °C.

The salted fish of 78.67 % (wb) average initial moisture content were dried to about 11.41% (wb) using hot drying air in the drying chamber of the solar dryer. The changes in the moisture content per amount of dry matter of fish with time are shown in Fig 4. The interruption of the lines in this figure represent the night period of the drying operation. Final drying level is realized in 33h in the solar dryer. Fig 5 clearly indicates that the drying rate in the solar dryer operating under natural convection could be much higher than natural operating-air sun drying. The final moisture contents represent moisture equilibrium between the sample and drying air under solar energy dryer conditions, beyond which any changes in the mass of sample could not occur.



**Fig 4** Variation of moisture content of fish flakes with drying time

It is apparent that drying rate decreases continuously with moisture content or drying time. There is no any constant rate drying period in these curves and all the drying operations are seen to occur in falling rate period. These results are in agreement with the earlier observations (Pangavhane et al., 1999; Tulasidas, Raghaquan, and Norris, 1993).



**Fig 5** Moisture content of open sun and close solar drying of fish flakes with time

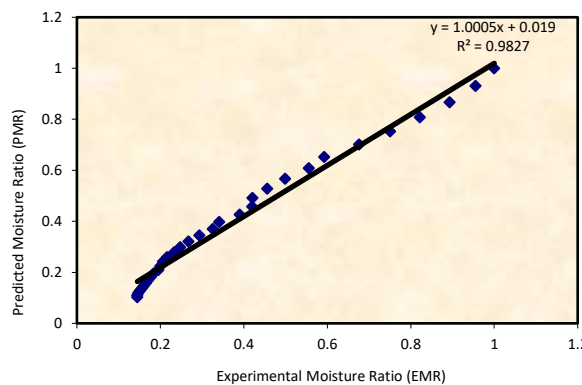
The moisture content data obtained from drying experiment were converted to the most useful moisture ratio expressions and then fitting computation with the drying time were carried on 5 drying models evaluated by previous workers. These models and the results of statistical analysis undertaken on them are given in table 1 and 2, respectively. The results have shown that the highest value of  $R^2$  and EF and lowest value of Chi-square ( $\chi^2$ ), and RMSE should be obtained with Lewis drying model. Thus, this model may be assumed to represent the solar drying behavior of fish in thin layer.

**Table 2** The calculated constants and coefficients and the validation statistical parameters of the tested five drying models

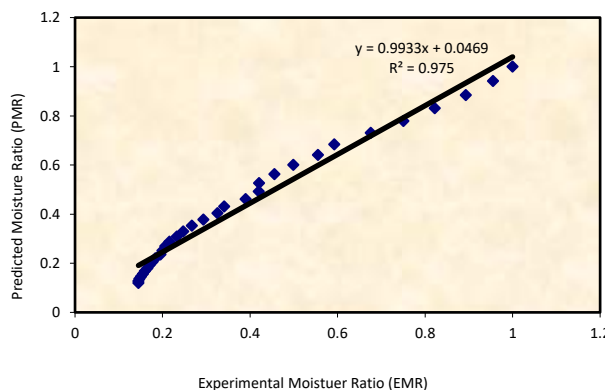
Model	Model constants	$R^2$	RMSE	$\chi^2$	EF%
Lewis	K = 0.071	0.9827	0.0391	0.0016	98
Page	K = 0.06 N = 1.03	0.9750	0.0603	0.0038	95
Henderson & Pabis	A = 0.871 K = 0.064	0.9763	0.0522	0.0030	97
Logarithmic	A = 0.99 K = 0.059 C = 0.012	0.9704	0.2572	0.0727	89
Diffusion	A = 0.34 K = 0.144 B = 0.355	0.9531	0.2727	0.0818	61

Validation of the established models was made by comparing the computed moisture contents with measured moisture contents in any particular drying run under certain conditions. The moisture content data

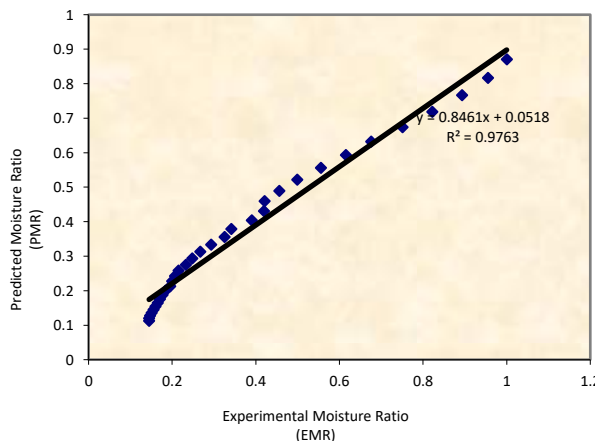
obtained every one hour interval by different drying air temperatures were converted to more useful moisture ratio expression and these measured moistures ratio from the experiment (MR<sub>exp.</sub>) were plotted versus predicted moisture (MR<sub>pre.</sub>) for each one of the 5 drying model equations Figs 5,6,7,8 and 9. The experimental data for Lewis drying model are generally banded around the straight line representing data found by computation, which indicates the suitability of the this model in describing drying behavior of fish.

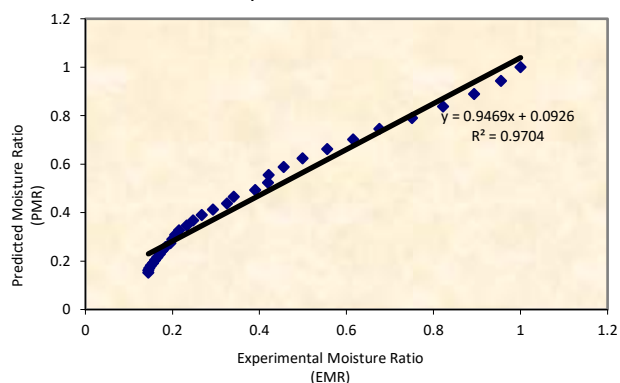
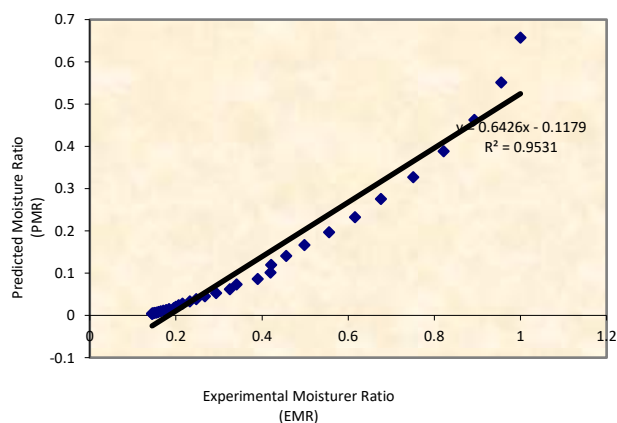


**Fig 5** Predicted MR by Lewis model versus experimental MR



**Fig 6** Predicted MR by Page model versus experimental MR



**Fig 7** Predicted MR by Henderson and Pabis model versus experimental MR**Fig 8** Predicted MR by logarithmic model versus experimental MR**Fig 9** Predicted MR by diffusion model versus experimental MR

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