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Some Forms of bT^{μ} - Normal Spaces in Supra Topological Spaces

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Abstract

In this paper, we introduce the new concept of bT^{μ} -normal space, almost bT^{μ} - normal space, mildly bT^{μ} - normal space and strongly bT^{μ} - normal space. We obtain the relationship between these normal spaces and studied the preservative theorem in supra topological spaces.

Keywords: *bT^{μ} - normal space; almost bT^{μ} - normal space; mildly bT^{μ} - normal space; strongly bT^{μ} - normal space.*

1. Introduction

In 1996, D.Andrijevic ([1]) have defined and studied b-open sets in topological space. The concept of supra topology were introduced by Mashhour.et.al ([10]) and also studied S-continuous maps and S*-continuous maps in Supra topological spaces. Also Mashhour.et.al ([10]) discussed that many results of topological spaces, whereas some become false. Also the authors remarked that the intersection of two supra open sets need not be supra open and also the intersection of an open set and supra open set need not be supra open. They were also introduced S-T_0, S-T_1,S-T_2,S-T_2'spaces and discussed their relationship with the topological spaces T_0,T_1,T_2,T_2' spaces. In 2008, R.Devi, S.Sampath kumar and M.Caldas ([2]) introduced supra α - open sets and sα -continuous functions and investigate some of the basic properties of this function. Sayed and Takashi Noiri ([11]) studied the approach of b-open set and supra b continuity in supra topological space in 2010 and also discussed about the relation between supra b-continuous maps and supra b-open maps. M.K.Singal and A.R. Singal ([12], introduced the concept of mildly normal spaces in topological spaces by using regular closed set. Normality plays an important role in the topological property. In recent years, many authors have studied several forms of normality ([4, 9]). Erdal Ekici and Takashi Noiri ([4]) introduced and studied the concept of generalization of normal, almost normal and mildly normal spaces and obtain the characterizations and the relationships of each normal space by using pre closed functions and also obtained the preservation theorems.

G.B.Navalagi ([9]) have examined the normality in context of new concept and also discussed that almost regular strongly compact space is almost p - normal and weakly p-regular P_1 paracompact space is mildly p - normal. The authors already have introduced a new concept bT^{μ}-closed set and studied about their continuity and irresoluteness.

The purpose of this paper is to introduce the new concept of bT^{μ}-normal space, almost bT^{μ}-normal space, mildly bT^{μ}-normal space and strongly bT^{μ}-normal space and studied their relations and properties of these spaces in supra topological spaces. We came to the conclusion that stronger form of supra normality is mildly bT^{μ}-normal space and weaker form of supra normality is strongly bT^{μ} -normal space.

2. Preliminaries

Definition 2.1[10, 11] A subfamily of τ of X is said to be a supra topology on X, if

(i) X,$\phi \in \mu$

(ii) if $A_i \in \mu$ for all i $\in$ J then $\cup A_i \in \mu$. The pair (X, τ) is called supra topological space.

The elements of τ are called supra open sets in (X, τ) and complement of a supra open set is called a supra closed set.

Definition 2.2 [10, 11]

(i) The supra closure of a set A is denoted by cl^{μ} (A) and is denoted as cl^{μ} (A) = $\cap$\{B: B is a supra closed set and A$\subseteq$ B.

(ii) The supra interior of a set A is denoted by int^{μ} (A) and is denoted as int^{μ} (A) = $\cup$\{B: B is a supra closed set and A $\supseteq$ B.

Definition 2.3 [10]

Let (X, τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4 [5]

A subset of a supra topological space (X, μ) is called bT^{μ}-closed set, if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is T^{μ}-open in (X, μ).

Definition 2.5 [4]

Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f: (X, τ) → (Y, σ) is called bT^{μ}-continuous, if $f^{-1}(V)$ is bT^{μ}-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition 2.6 [8]

A function f: (X, τ) → (Y, σ) is called contra bT^{μ}-continuous, if $f^{-1}(V)$ is bT^{μ}-closed in (X, τ) for every supra open set V of (Y, σ).

Definition 2.7 [10]

A function f: (X, τ) → (Y, σ) is called S^{*}- continuous if $f^{-1}(V)$ is supra closed in (X, τ) for every supra open set V of (Y, σ).

Definition 2.8 [6]

A map f: (X, τ) → (Y, σ) is said to be a bT^{μ}-closed map (bT^{μ} -open map) if the image f (A) is bT^{μ} -closed (bT^{μ} -open) in (Y, σ) for each supra closed (supra open) set A in (X, τ).

Definition 2.9 [7]

Let (X, τ) and (Y, σ) be two topological spaces and be an associated supra topology with μ. A function f: (X, τ) → (Y, σ) is called strongly bT^{μ} - continuous, if $f^{-1}(V)$ is supra closed in (X, τ) for every bT^{μ} -closed set V of (Y, σ).

Definition 2.10 [3]

A function f: (X, τ) → (Y, σ) is called rc-continuous if inverse image of each regular closed set of Y is regular closed set in X.

Definition 2.11 [12]

A space X is called supra normal (resp. mildly normal) if for any pair of disjoint supra closed (resp. supra regular closed) subsets A and B of X, then there exist disjoint supra open sets U and V such that $A \subseteq U$, $B \subseteq V$.

Definition 2.12 [9]

A space X is called supra almost normal if for any disjoint supra closed set A and supra regular closed set B of X, there exist disjoint supra open sets U and V such that $A \subseteq U$, $B \subseteq V$.

Definition 2.13 [13]

A subset A of a space X is said to be supra regular open, if $A = int^{\mu}(cl^{\mu}(A))$. The complement of supra regular open set is supra regular closed.

Definition 2.14

A function f: (X, τ) → (Y, σ) is called supra rc- continuous if inverse image of each supra regular closed set of Y is supra regular closed set in X.

3. bT^{μ} -Normal Space

Definition 3.1

A space X is said to be bT^{μ}-normal if for any pair of disjoint supra closed sets A and B, their exist, disjoint bT^{μ}-open sets U and V such that $A \subset U$ and $B \subset V$.

Remark 3.2

The following implication holds for a supra topological space (X, τ).

Supra normal → bT^{μ}-normal but the converse is not true by the following example.

Example 3.3 Let X = {a, b, c, d} with τ = {X,ϕ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}}.Then the space (X, τ) is bT^{μ} -normal space but not supra normal space. Since supra closed set A= {a} and B = {b} is not proper disjoint subset of supra open set.

Theorem 3.4

For a space (X, τ) the following are equivalent

(i) (X, τ) is bT^{μ} - normal space.

(ii) For every pair of supra open sets U and V whose union is X, there exist bT^{μ} - closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(iii) For every supra closed set C and every supra open set D containing C there exist a bT^{μ} –open set U such that $C \subset U \subset bT^{\mu}\text{-}cl(U) \subset D$.

Proof

(i)→(ii) Let U and V be any supra open subsets of a bT^{μ} -normal space X such that $U \cup V = X$. Then X/U and X/V are disjoint supra closed subsets of X. By normality bT^{μ} -normality of X, there exist disjoint bT^{μ} - open sets U_1 and V_1 of X such that $X/U \subset U_1$ and $X/V \subset V_1$. Let $A = X/U_1$ and $B = X/V_1$. Then A and B are bT^{μ} -closed subsets of X such that $A \subset U$ and $B \subset V$ and $A \cup B = X$.

(ii) → (iii) Let C be a supra closed and D be supra open set containing C. Then X/C and D are supra open sets whose union is X. Then by (ii) there exist bT^{μ} - closed sets M_1 and M_2 such that $M_1 \subset X/C$ and $M_2 \subset D$ and $M_1 \cup M_2 =$

X. Then $C \subset X/M_1$, $X/D \subset X/M_2$ and $(X/M_1) \cap (X/M_2) = \phi$. Let $U = X/M_1$ and $V = X/M_2$. Then U and V are disjoint bT^μ - open sets such that $C \subset U \subset X/V \subset D$. As X/V is bT^μ - closed set, we have bT^μ - $cl(U) \subset X/V$ and $C \subset U \subset bT^\mu$ -$cl(U) \subset D$.

(iii)→ (ii) Let C_1 and C_2 be any two disjoint supra closed sets of X. Put $D = X/C_2$, then $C_2 \cap D = \phi$. $C_1 \subset D$, where D is a supra open set. Then by (iii), there exist a bT^μ -open set U of X such that $C_1 \subset U \subset bT^\mu$ - $cl(U) \subseteq D$. It follows that $C_2 \subset X/ bT^\mu$ - $cl(U)$ = V say then V is bT^μ - open and $U \cap V = \phi$. Hence C_1 and C_2 are separated by bT^μ - open sets U and V. Therefore X is bT^μ - normal.

Definition 3.5

A function f: (X, τ) → (Y, σ) is called strongly bT^μ -open if the image f(A) is bT^μ - open in Y for each bT^μ - open set A in X.

Theorem 3.6

A supra open subspace of a bT^μ - normal space is bT^μ - normal.

Proof Let Y be supra open subspace of a bT^μ - normal space X. Let A and B be disjoint supra closed subsets of Y. As Y is supra open, A and B are supra closed sets of X. By bT^μ -normality of X, there exist disjoint bT^μ -open sets U and V in X such that $A \subset U$ and $B \subset V$, $U \cap Y$ and $V \cap Y$ are supra open in Y, By ([5]) , we know that every supra open set is bT^μ -open. Therefore $U \cap Y$ and $V \cap Y$ are bT^μ - open in Y such that $A \subset U \cap Y$ and $B \subset V \cap Y$. Hence Y is bT^μ -- normal.

Theorem 3.7

A function f: (X, τ) → (Y, σ) is strongly bT^μ -open map if and only if for each subset B in Y and for each bT^μ -open set U in X containing $f^{-1}(B)$ there exist a bT^μ - open set V containing B such that $f^{-1}(V) \subset U$.

Proof ⇒ Suppose that f is strongly bT^μ -closed. Let $B \subseteq Y$, U be a bT^μ -open set of (X, τ) such that $f^{-1}(V) \subset U$. Then $V = (f(U^c)^c)$ is a bT^μ -open set such that $f^{-1}(V) \subset U$.

⇐ Conversely, let A be a bT^μ -closed set of (X,τ).Then $f^{-1}((f(U^c))^c) \subset A^c$ and A^c is bT^μ -open. By assumption, there exist a bT^μ - open set V of (Y,σ) such that $(f(A))^c \subset V$ and $f^{-1}(V) \subset A^c$ and so $A \subset (f^{-1}(V))^c$. Hence $V^c \subset f(A) \subset f(f^{-1}(V))^c \subset V^c$. Thus $f(A) = V^c$. Since V^c is bT^μ - closed f (A) is bT^μ - closed and therefore f is strongly bT^μ - closed map.

Theorem 3.8

A function f: (X, τ) → (Y, σ) is strongly bT^μ - open map, S^*- continuous function from a bT^μ - normal space X onto a space Y, then Y is bT^μ - normal.

Proof Let A and B are disjoint supra closed sets in Y. Then by S^*- continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are supra closed sets. Since X is bT^μ - normal, then there exist disjoint supra open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Therefore by the strongly bT^μ - open map, $A \subset f(U)$ and $B \subset f(V)$, f (U) and f(V) are bT^μ - open sets in Y. Hence Y is bT^μ - normal.

Theorem 3.9

A function f: (X, τ) → (Y, σ) is strongly bT^μ - open map, S^*- continuous function from a supra normal space X onto a space Y, then Y is bT^μ - normal.

Proof Let C and D are disjoint supra closed sets in Y. Then by S^*- continuous, $f^{-1}(C)$ and $f^{-1}(D)$ are supra closed sets. Since X is supra normal, then there exist disjoint bT^μ –Open sets U and V. By [5] we know that every supra open closed set is bT^μ - open. Therefore U and V are bT^μ - open sets such that $f^{-1}(C) \subset U$ and $f^{-1}(D) \subset V$. Therefore by the strongly bT^μ - open map, $A \subset f(U)$ and $B \subset f(V)$, f(U) and f(V) are bT^μ - open sets in Y. Hence Y is bT^μ - normal.

Theorem 3.10

A function f: (X, τ) → (Y, σ) is S^*- continuous, closed injection and (Y,σ) is bT^μ -normal then (X, τ) is bT^μ - normal.

Proof Let A and B be any two disjoint supra closed sets of (X, τ). Since f is supra closed map, f (A) and f (B) are disjoint supra closed subset of Y. Since Y is bT^μ - normal, f (A) and f (B) are separated by disjoint bT^μ - open sets U and V respectively. Hence $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{1}(V) = \phi$. Thus X is bT -normal.

Theorem 3.11

A function f: (X, τ) → (Y, σ) is a contra bT^μ - continuous, closed injection and Y is bT^μ - normal then X is bT^μ - normal.

Proof Let A_1 and A_2 be a disjoint supra closed subsets of X. Since f is supra closed injective, $f(A_1)$ $f(A_1)$ and $f(A_2)$ are disjoint supra closed subsets of Y. Since Y is bT^μ -normal, f (A_1) and $f(A_2)$ are separated by disjoint bT^μ -open sets V_1 and V_2 respectively. Hence $A_i \subset f^{-1}(V_i)$,
$f^{-1}(V_i) \in bT^\mu$ O(X) for i = 1,2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Hence X is bT^μ - normal.

Theorem 3.12

A function f: (X, τ) → (Y, σ) is a contra bT^μ - continuous, closed injection and Y is supra ultra normal then X is bT^μ - normal.

Proof Let A_1 and A_2 be a disjoint supra closed subsets of X. Since f is supra closed injective, f (A_1) and f (A_2) are disjoint supra closed subsets of Y. Since Y is supra ultra normal, f (A_1) and f (A_2) are separated by disjoint supra clopen sets V_1 and V_2 respectively. Hence $A_i \subset f^{-1}(V_i)$, $f^{-1}(V_i) \in bT^\mu$ O(X) for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Hence X is bT^μ - normal.

Almost bT^{μ} -Normal Space

Definition 4.1

A space X is said to be almost bT^{μ} - normal if for any supra closed set A and any supra regular closed set B such that $A\cap B =\varphi$, their exist disjoint bT^{μ} -open sets U and V such that $A\subset U$ and $B\subset V$.

Remark 4.2 The following implication holds for a supra topological space (X, τ).
Supra almost normal $\rightarrow$ almost bT^{μ} - normal but the converse is not true by the following example.

Example 4.3 Let X = {a, b, c, d, e} with τ = {X, φ, {b, c}, {b, e}, {b, c, e}, {a, d}, {a, b, c, d}, {a, b, d, e}}.The supra closed sets are { X, φ, {c}, {e}, {a, d}, {a, d, e}, {a, c, d}, {b, c, e} and the supra regular closed sets are {X, φ, {a, d}, {b, c, e}}. The bT^{μ} –open sets are {X, φ, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, c}, {b, d}, {c, d}, {c, e}, {d, e}, {a, b, d}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, e}, {c, d, e}, {a, b, c, e}, {a, c, d, e}, {b, c, d, e}}. Then the space (X, τ) is almost bT^{μ} - normal space but not supra almost normal space. Since supra closed set A= {e} and supra regular closed set B = {a, d} is not proper disjoint subset of supra open set.

Theorem 4.4

For a space (X, τ) the following are equivalent
(i) (X, τ) is almost bT^{μ} - normal space.
(ii) For any pair of supra open sets U and V, one of which is supra open and the other is supra regular open, $U\cup V = X$, there exist bT^{μ} - closed sets A and B such that $A\subset U$, $B\subset V$ and $A\cup B = X$.
(iii) For any supra closed set A and any supra regular open set B containing A there exist a bT^{μ} - open set U such that $A\subset U\subset bT^{\mu}\text{-}cl(U) \subset B$.

Proof
(i) →(ii) Let U be an supra open set and V be an supra regular open set in an almost bT^{μ} - normal space X such that $U \cup V = X$. Then X/U is a supra closed set and X/V is a supra regular closed set with $X/U\cap X/V =\phi$. By almost bT^{μ} - normality of X, there exist disjoint bT^{μ} - open sets U_1 and V_1 of X such that $X/U \subset U_1$ and $X/V\subset V_1$.Let A = X/U_1 and B = X/V_1. Then A and B are bT^{μ} - closed subsets of X such that $A\subset U$ and $B\subset V$ and $A \cup B = X$.

(ii) →(iii) Let A be a supra closed set and B be a supra regular open set containing A. Then X/A is supra open set such that $X/A\cup B = X$. Then by (ii) there exist bT^{μ} - closed sets M_1 and M_2 such that $M_1\subset X/A$ and $M_2\subset B$ and $M_1\cup M_2 = X$. Then $A \subset X/M_1$, $X/B \subset X/M_2$ and $(X/M_1)\cap\backslash(X/M_2) =\phi$. Let U = X/M_1 and V = X/M_2. Then U and V are disjoint bT^{μ} -open sets such that $A\subset U \subset X/V\subset B$. As X/V is bT^{μ} - closed set, we have bT^{μ} - cl(U) $\subset X/V$ and $A\subset U\subset bT^{\mu}$ - cl(U) $\subset B$.

(iii)→(i) Let K_1 and K_2 be any two disjoint supra closed and supra regular closed set respectively. Put B = X/K_2, then $K_2\cap B =\phi$. $K_1\subset B$, where B is a supra regular open set. Then by (iii) it follows that $K_2\subset X/ bT^{\mu}$ - cl(U) = V say then V is bT^{μ} - open and $U\cap V =\phi$. Hence K_1 and K_2 are separated by bT^{μ} - open sets U and V. Therefore X is almost bT^{μ} -normal.

Theorem 4.5

A function f: (X, τ) $\rightarrow$ (Y, σ) is strongly bT^{μ} - open map, S^*-continuous function supra rc-continuous from a bT - normal space X onto a space Y, then Y is almost bT^{μ} - normal.
Proof Let M_1 and M_2 are disjoint supra closed and supra regular closed sets in Y respectively. Then by S^*-continuous, $f^{-1}(M_1)$ is supra closed and by supra rc-continuous and $f^{-1}(M_2)$ are supra regular closed sets. Since X is bT^{μ} - normal, then there exist disjoint supra bT^{μ} -open sets U and V such that $f^{-1}(M_1) \subset U$ and $f^{-1}(M_2) \subset V$. Therefore by the strongly bT^{μ} - open map, $M_1\subset f(U)$ and $M_2\subset f(V)$, f (U) and f(V) are bT^{μ} - open sets in Y. Hence Y is almost bT^{μ} -normal.

5. Mildly bT^{μ}-Normal Space

Definition 5.1 A space X is said to be mildly bT^{μ}-normal if for any pair of disjoint
Supra regular closed sets A and B of X, their exist disjoint bT^{μ} -open sets U and V such that $A\subset U$ and $B\subset V$.

Example 5.2 Let X = {a, b, c, d} with τ = {X, φ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}}. The supra regular closed sets are {X, φ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}}. Let A = {b} and B = {d} be disjoint supra regular closed sets. The bT^{μ} -open sets are {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Let the disjoint bT^{μ} -open sets U = {a, b} and V = {c, d} such that $A\subset U$ and $B\subset V$. Therefore the space (X, τ) is mildly bT^{μ} - normal space.

Theorem 5.3

For a space X the following st atement is equivalent
(i) X is mildly bT^{μ} - -normal
(ii) For any pair of regular open sets U_1 and U_2 whose Union is X, there exist bT^{μ} -closed sets G and H, such that $G\subset U_1$, $H\subset U_2$ and $G \subset U_1\cup H\subset U_2 = X$.
(iii) For any regular closed set A and every regular open set B containing A, there exist a bT^{μ} - open set V such that $A\subset V\subset bT^{\mu}$ -cl(V)$\subset$ B.
(iv) For every point of disjoint supra regular closed sets there exist bT^{μ} - open set $U_1$and U_2 such that $A\subset U_1$, $B\subset U_2$ and bT -cl$(U_1)\cap$bT -cl$(U_2) = \phi$.

Proof (i) ⇒ (ii) Let U_1 and U_2 be a regular open sets in mildly bT^{μ} -normal space X such that $U_1\cup U_2$= X. Then X/U_1 and X/U_2 are supra regular closed set with $X/U_1\cap X/U_2 = \phi$

By mildly bT^{μ} - normality of X, there exist disjoint bT^{μ} - open sets V_1 and V_2 such that $X/U_1 \subset V_1$ and $X/U_2 \subset V_2$. Let $G = X/U_1$ and $H = X/U_2$. Then G and H are bT^{μ} -closed sets such that $G \subset U_1$, $H \subset U_2$ and $G \cup H = X$.

(ii) ⇒(iii) Let A be a supra regular closed set and supra regular open set containing A. Then X/A and B supra regular open sets whose union is X. Then by (ii) there exist bT^{μ} - closed sets A_1 and A_2 and $A_1 \cup A_2 = X$. Then $A \subset X/A_1$, $X/B \subset X/A_2$ and $X/A_1 \cap X/A_2 = \phi$. Let $V = X/A_1$ and $W = X/A_2$. Then V and W are disjoint bT^{μ} -open sets such that $A \subset V \subset X/W \subset B$. As X/W is bT^{μ} - closed set. We have bT^{μ} -cl(V) $\subset X/W$ and $A \subset V \subset bT^{\mu}\text{-cl}(V) \subset B$.

(iii) ⇒(iv) Let F_1 and F_2 be any two disjoint regular closed sets of X. Then X/F_1 and X/F_2 are regular open sets such that $(X/F_1) \cup (X/F_2 = X$. Then by (iii) there exist a bT^{μ} - open set W_1 and W_2 such that $W_1 \subset X/F_1$ and $W_2 \subset X/F_2$ and $W_1 \cup W_2 = X$. Then $F_1 \subset X/W_1$, $F_2 \subset X/W_2$ and $X/W_1 \cap X/W_2 = \phi$. Let $U_1 = X/W_1$ and $U_2 = X/W_2$. Therefore, $F_1 \subset U_1$ and $F_2 \subset U_2$. By condition (iii) $F_1 \subset U_1 \subset bT^{\mu}\text{-cl}(U_1) \subseteq F_2$. Obvisiouly, $bT^{\mu}\text{-cl}(U_1) \cap bT^{\mu}\text{-cl}(U_1) = \phi$

(iv) ⇒(i) Let A and B are disjoint supra regular closed. By the condition (iv), there exist a bT^{μ} -open sets U and V such that $A \subset U$ and $B \subset V$.

Theorem 5.4

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} - open map and supra rc-continuous of a mildly bT^{μ} - normal space X onto a space Y, then Y is mildly bT^{μ} - normal.

Proof Let A_1 and A_2 be disjoint supra regular closed sets in Y. Then by supra rc- continuous, $f^{-1}(A_1)$ and $f^{-1}(A_2)$ are supra regular closed sets in X. Since X is mildly bT^{μ} - normal, there exist disjoint supra bT^{μ} - open sets U and V such that $f^{-1}(A_1) \subset U$ and $f^{-1}(A_2) \subset V$. Therefore by the strongly bT^{μ} - open map, $A_1 \subset f(U)$ and $A_2 \subset f(V)$, f(U) and f(V) are bT^{μ} -open sets in Y. Hence Y is mildly bT^{μ} - normal.

Theorem 5.5

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} - open map and supra rc-continuous of an almost bT^{μ} - normal space X onto a space Y, then Y is mildly bT^{μ} - normal.

Proof Let A and B be disjoint supra regular closed sets in Y. Then by supra rc- continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are supra regular closed sets in X. Since X is almost bT^{μ} - normal, there exist disjoint bT^{μ} -open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Therefore by the strongly bT^{μ} -open map, $A \subset f(U)$ and $B \subset f(V)$, f(U) and f(V) are bT^{μ} - open sets in Y. Hence Y is mildly bT^{μ} - normal.

Theorem 5.6

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} - open map and S^*-continuous surjection from a bT^{μ} - normal space X onto a space Y, then Y is mildly bT^{μ} -normal.

Proof Let A and B be disjoint supra regular closed sets in Y. We know that every supra regular closed set is supra closed set. Therefore by S^*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are supra regular closed sets in X. Since X is bT^{μ} - normal, there exist disjoint bT^{μ} - open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Therefore by the strongly bT^{μ} -open map, $A \subset f(U)$ and $B \subset f(V)$, f(U) and f(V) are bT^{μ} -open sets in Y. Hence Y is mildly bT^{μ} - normal.

6. Strongly bT^{μ}-Normal Space

Definition 6.1

A space X is said to be strongly bT^{μ} - normal if for any pair of disjoint bT^{μ} - closed sets A and B, their exist disjoint bT^{μ} - open sets U and V such that $A \subset U$ and $B \subset V$.

Example 6.2 Let X = {a, b, c, d, e} with τ = {X, φ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}. The bT^{μ} – closed sets are {X, φ, {b}, {c}, {d}, {e}, {a, b}, {a, c},{a, e}, { b, d}, {b, e}, {c, d}, {c, e},{d, e}, {a, b, c}, {a, b, d}, {a, c, d}, {a, b e}, {a, c, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}, {a, d, e}, {a, b, c, e}, {a, c, d, e}, {a, b, d, e}, {b, c, d, e}} and bT^{μ} – open sets are {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {a, e}, {b, c}, { b, d}, {b, e}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {a, b e}, {a, c, e}, {b, c, d}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, c, e}, {a, c, d, e}, {a, b, d, e}, {b, c, d, e}}. Let the disjoint bT^{μ} – closed sets A = {e} and B = {a, c}, their exist, disjoint bT^{μ} – open sets U = {b, e} and V = {a, c, d} such that $A \subset U$ and $B \subset V$. Therefore the space (X, τ) is strongly bT^{μ} - normal space.

Theorem 6.3

Let (X, τ) be a supra topological space. Then the following statements are equivalent.

(i) X is strongly bT^{μ} -normal

(ii) For each bT^{μ} -closed set A and for each bT^{μ} -open set B containing A, there exist a bT^{μ} - open set U_1 containing A such that $bT^{\mu}\text{-cl}(U_1) \subset B$.

(iii) For each pair of disjoint bT^{μ} - closed sets A and B in (X, τ), there exist a bT^{μ} -open set U_1 containing A such that $bT^{\mu}\text{-cl}(U_1) \cap B = \phi$.

(iv) For each pair of disjoint bT^{μ} - closed sets A and B in (X, τ), there exist bT^{μ} - open sets U_1 containing A and U_2 containing B such that $bT^{\mu}\text{-cl}(U_1) \cap bT\text{-cl}(U_2) = \phi$.

Proof (i) ⇒ (ii) Let A be a bT^{μ} - closed set and B be a bT^{μ} - open set such that $A \subset B$. Then $A \cap B^c = \phi$. Since X is strongly bT^{μ} -normal, there exist bT^{μ} -open set U_1 and U_2 such that $A \subset U_1$, $B^c \subset U_2$ and $U_1 \cap U_2 = \phi$, which implies $bT^{\mu}\text{-cl}(U_1) \cap U_2 = \phi$. Now $bT^{\mu}\text{-cl}(U_1) \cap B^c \subset bT^{\mu}\text{-cl}(U_1) \cap U_2 = \phi$ and so $bT^{\mu}\text{-cl}(U_1) \subset B$.

(ii)⇒(iii) Let A and B be disjoint bT^{μ} - closed sets in (X, τ).Since $A \cap B = \phi$, $A \subset B^c$ and B^c is bT -open . By (ii), there exist an bT^{μ} -open set U_1 containing A such that $bT^{\mu}\text{-cl}(U_1) \subset B^c$ and so $bT\text{-cl}(U_1) \cap B = \phi$.

(iii)⇒(iv) Let A and B be any two disjoint bT^{μ} - closed sets of (X, τ). Then by (iii), there exist bT^{μ} - open set U_1 containing A such that bT^{μ} -cl(U_1) ∩ B^c =ϕ . Since B and bT^{μ} - cl(U_1) are disjoint bT^{μ} - closed sets in (X, τ). Therefore again by (iii), there exist a bT^{μ} - open set U_2 containing B such that bT^{μ} -cl(U_1) ∩ bT^{μ} -cl(U_2)= ϕ.

(iv) ⇒ (i) Let A and B be any two disjoint bT^{μ} - closed sets of (X, τ). By (iv), there exist bT^{μ} - open set U_1 containing A and U_2 containing B such that bT^{μ} - cl(U_1) ∩ bT^{μ} -cl(U_2)=ϕ., we have $U_1 \cap U_2$ =ϕ and thus (X, τ) is bT^{μ} - normal.

Theorem 6.4

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} -open map, S^* - continuous function surjection from a strongly bT^{μ} - normal space X onto a space Y, then Y is bT^{μ} -normal.

Proof Let A and B be disjoint supra closed sets in Y. Then by S^* - continuous, f^{-1}(A) and f^{-1}(B) are supra closed sets. Since X is strongly bT^{μ} - normal, then there exist disjoint bT^{μ} - open sets U and V such that f^{-1}(A) ⊂U and f^{-1}(B) ⊂V. Therefore by the strongly bT^{μ} - open map, A⊂ f (U) and B ⊂ f (V), f (U) and f(V) are bT^{μ} -open sets in Y. Hence Y is bT^{μ} -normal.

Theorem 6.5

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} - open map, strongly bT^{μ} -continuous function surjection from a strongly bT^{μ} - normal space X onto a space Y, then Y is almost bT^{μ} - normal.

Proof Let A be supra closed set and B be supra regular closed sets in Y. Then by strongly bT^{μ} - continuous, f^{-1}(A) and f^{-1}(B) are supra closed sets in X. We know that every supra closed set is bT^{μ} - closed set. Therefore f^{-1}(A) and f^{-1} (B) are bT^{μ} - closed set. Since X is strongly bT^{μ} - normal, then there exist disjoint bT^{μ} - open sets U and V such that f^{-1}(A) ⊂ U and f^{-1}(B) ⊂ V. Therefore by the strongly bT^{μ} - open map, A ⊂ f (U) and B⊂ f (V), f (U) and f (V) are bT^{μ} - open sets in Y. Hence Y is almost bT^{μ} -normal.

Theorem 6.6

A function f: (X, τ) → (Y, σ) is supra contra continuous function, bT^{μ} -closed map, injection and Y is strongly bT^{μ} - normal space then X is bT^{μ} -normal.

Proof Let A_1 and A_2 be disjoint supra closed subsets in X. Since f is bT^{μ} - closed map injective, f (A_1) and f(A_2) are bT^{μ} –closed subset of Y. Since Y is strongly bT^{μ} - normal,there exist, f(A_1) and f(A_2) are separated by disjoint bT^{μ} - open sets V_1 and V_2 respectively. Hence A_i⊂ $f^{-1}(V_i)$, $f^{-1}(V_i) \in bT^{\mu}O(X)$ for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}V_2)$ =ϕ. Hence X is bT^{μ} - normal.

Theorem 6.7

A function f: (X, τ) → (Y, σ) is strongly bT^{μ} -open map, strongly bT^{μ} -continuous function surjection from a strongly bT^{μ} -normal space X onto a space Y, then Y is mildly bT^{μ} -normal.

Proof Let A and B be disjoint supra regular closed sets in Y. Then by strongly bT^{μ} -continuous, f^{-1}(A) and f^{-1} (B) are supra closed sets in X. Since X is strongly bT^{μ} -normal, then there exist disjoint bT^{μ} - open sets U and V such that f^{-1} (A) ⊂U and f^{-1} (B) ⊂ V. Therefore by the strongly bT^{μ} - open map, A⊂ f (U) and B⊂ f (V), f (U) and f(V) are bT^{μ} -open sets in Y. Hence Y is mildly bT^{μ} -normal space.

Remark 6.8 The following implication holds for a supra topological space (X, τ).

Mildly bT^{μ} - normal space → almost bT^{μ} -normal space → bT^{μ} -normal → strongly bT^{μ} - normal spaces but the converse is not true by the following example.

Example 6.9 Let X = {a, b, c, d, e} with τ = {X, φ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}. The supra closed sets are {X, φ, bT^{μ} – closed sets are {X, φ, {b}, {c}, {d}, {e}, {a, b}, {a, c},{a, e}, { b, d}, {b, e}, {c, d}, {c, e},{d, e}, {a, b, c}, {a, b, d}, {a, c, d}, {a, b e}, {a, c, e}, {b, c, d}, {b, c, e}, {b, d, e}, {c, d, e}, {a, d, e}, {a, b, c, e}, {a, c, d, e}, {a, b, d, e}, {b, c, d, e}} and bT^{μ} –open sets are {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {a, e}, {b, c}, { b, d}, {b, e}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {a, b e}, {a, c, e}, {b, c, d}, {b, d, e}, {c, d, e}, {a, b, c, d}, {a, b, c, e}, {a, c, d, e}, {a, b, d, e}, {b, c, d, e}}. Let the disjoint bT^{μ} –closed sets A = {d} and B = {a, c}, their exist, disjoint bT^{μ} –open sets U = {d, e} and V = {a, b, c} such that A⊂U and B ⊂ V. Therefore the space (X, τ) is strongly bT^{μ} - normal space but not bT^{μ} -normal, almost bT^{μ} -normal and mildly bT^{μ} - normal spaces. Since, there is no disjoint A and B supra closed set and supra regular closed set in (X, τ).

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