

Connected Total Dominating Sets and Connected Total Domination Polynomials of Triangular Ladders

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Abstract

Let G be a simple connected graph of order n . Let $D_{ct}(G, i)$ be the family of connected total dominating sets of G with cardinality i . The polynomial $D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^n d_{ct}(G, i) x^i$ is called the connected total domination polynomial of G . In this paper, we study some properties of connected total domination polynomials of the Triangular Ladder TL_n . We obtain a recursive formula for $d_{ct}(TL_n, i)$. Using this recursive formula, we construct the connected total domination polynomial $D_{ct}(TL_n, x) = \sum_{i=n-1}^{2n} d_{ct}(TL_n, i) x^i$, of TL_n , where $d_{ct}(TL_n, i)$ is the number of connected total dominating sets of TL_n with cardinality i and some properties of this polynomial have been studied.

Keywords: Triangular Ladder, Connected Total Dominating Set, Connected Total Domination Number, Connected Total Domination Polynomial.

1. Introduction

Let $G = (V, E)$ be a simple connected graph of order n . For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. The maximum degree of the graph G is denoted by $\Delta(G)$ and the minimum degree is denoted by $\delta(G)$.

A set S of vertices in a graph G is said to be a total dominating set if every vertex $v \in V$ is adjacent to an element of S .

A total dominating set S of G is called a connected total dominating set if the induced subgraph $\langle S \rangle$ is connected.

The minimum cardinality taken over all connected total dominating sets S of G is called the connected total domination number of G and is denoted by $\gamma_{ct}(G)$.

A connected total dominating set with cardinality $\gamma_{ct}(G)$ is called γ_{ct} -set. We denote the set $\{1, 2, \dots, 2n-1, 2n\}$ by $[2n]$, throughout this paper.

2. Connected total dominating sets of triangular ladders

Consider two paths $[u_1 u_2 \dots u_n]$ and $[v_1 v_2 \dots v_n]$. Join each pair of vertices u_i, v_i and u_{i+1}, v_i , $i = 1, 2, \dots, n$. The resulting graph is a Triangular Ladder.

Let TL_n be a Triangular ladder with $2n$ vertices. Label the vertices of TL_n as given in Figure 1.

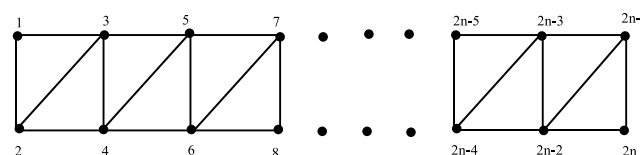


Figure 1

Triangular Ladder TL_n

Then, $V(TL_n) = \{1, 2, 3, \dots, 2n-3, 2n-2, 2n-1, 2n\}$ and $E(TL_n) = \{(1, 3), (3, 5), (5, 7), \dots, (2n-5, 2n-3), (2n-3, 2n-1), (2, 4), (4, 6), (6, 8), \dots, (2n-4, 2n-2), (2n-2, 2n), (1, 2), (3, 4), (5, 6), \dots, (2n-3, 2n-2), (2n-1, 2n), (2, 3), (4, 5), (6, 7), \dots, (2n-4, 2n-3), (2n-2, 2n-1)\}$.

For the construction of the connected total dominating sets of the Triangular Ladders TL_n , we need to investigate the connected total dominating sets of $TL_n - \{2n\}$. In this section, we investigate the connected total dominating sets of TL_n with cardinality i . We shall find the recursive formula for $d_{ct}(TL_n, i)$.

Lemma 2.1[7].

$$\gamma_{ct}(P_n) = n - 2.$$

Lemma 2.2

For every $n \in \mathbb{N}$ and $n \geq 4$,

- (i) $\gamma_{ct}(TL_n) = n - 1$.
- (ii) $\gamma_{ct}(TL_n - \{2n\}) = n - 2$.
- (iii) $D_{ct}(TL_n, i) = \phi$ if and only if $i < n - 1$ or $i > 2n$.
- (iv) $D_{ct}(TL_n - \{2n\}, i) = \phi$ if and only if $i < n - 2$ or $i > 2n - 1$.

Proof

- (i) Clearly $\{3, 5, 7, 9, \dots, 2n - 1\}$ is a minimum connected total dominating set for TL_n . If n is even or odd it contains $n - 1$ elements. Hence, $\gamma_{ct}(TL_n) = n - 1$.
- (ii) Clearly $\{3, 5, 7, 9, \dots, 2n - 3\}$ is a minimum connected total dominating set for $TL_n - \{2n\}$. If n is even or odd it contains $n - 2$ elements. Hence, $\gamma_{ct}(TL_n - \{2n\}) = n - 2$.
- (iii) follows from (i) and the definition of connected total dominating set.
- (iv) follows from (ii) and the definition of connected total dominating set.

Lemma 2.3

- (i) If $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$ and $D_{ct}(TL_{n-2}, i - 1) = \phi$, then $D_{ct}(TL_{n-1}, i - 1) = \phi$.
- (ii) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-2}, i - 1) \neq \phi$, then $D_{ct}(TL_{n-1}, i - 1) \neq \phi$.
- (iii) If $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$ and $D_{ct}(TL_{n-1}, i - 1) = \phi$, then $D_{ct}(TL_n, i) = \phi$.
- (iv) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-1}, i - 1) \neq \phi$, then $D_{ct}(TL_n, i) \neq \phi$.
- (v) If $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, and $D_{ct}(TL_{n-1}, i - 1) = \phi$, then $D_{ct}(TL_n, i) \neq \phi$.

Proof

- (i) Since, $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$ and $D_{ct}(TL_{n-2}, i - 1) = \phi$, by Lemma 2.2 (iii) & (iv), we have, $i - 1 < n - 2$ or $i - 1 > 2n - 1$, $i - 1 < n - 3$ or $i - 1 > 2n - 3$ and $i - 1 < n - 3$ or $i - 1 > 2n - 4$. Therefore, $i - 1 < n - 3$ or $i - 1 > 2n - 1$. Therefore, $i - 1 < n - 2$ or $i - 1 > 2n - 2$ holds. Hence, $D_{ct}(TL_{n-1}, i - 1) = \phi$.

- (ii) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$,

$D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-2}, i - 1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have, $n - 2 \leq i - 1 \leq 2n - 1$, $n - 3 \leq i - 1 \leq 2n - 3$ and $n - 3 \leq i - 1 \leq 2n - 4$. Suppose, $D_{ct}(TL_{n-1}, i - 1) = \phi$. Then, by Lemma 2.2 (iii), we have, $i - 1 < n - 2$ or $i - 1 > 2n - 2$. If $i - 1 < n - 2$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction. If $i - 1 > 2n - 2$, then $i - 1 > 2n - 3$ holds, which implies $D_{ct}(TL_{n-1} - \{2n - 2\}, i - 1) = \phi$, a contradiction. Therefore, $D_{ct}(TL_{n-1}, i - 1) \neq \phi$.

(iii) Since, $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$ and $D_{ct}(TL_{n-1}, i - 1) = \phi$, by Lemma 2.2 (iii) & (iv), we have, $i - 1 < n - 2$ or $i - 1 > 2n - 1$ and $i - 1 < n - 2$ or $i - 1 > 2n - 2$. Therefore, $i - 1 < n - 2$ or $i - 1 > 2n - 1$.

Therefore, $i < n - 1$ or $i > 2n$.

Hence, $D_{ct}(TL_n, i) = \phi$.

(iv) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-1}, i - 1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have, $n - 2 \leq i - 1 \leq 2n - 1$ and $n - 2 \leq i - 1 \leq 2n - 2$. Suppose, $D_{ct}(TL_n, i) = \phi$, then, by Lemma 2.2 (iii), we have $i < n - 1$ or $i > 2n$. Therefore, $i - 1 < n - 2$ or $i - 1 > 2n - 1$. If $i - 1 < n - 2$, then $D_{ct}(TL_{n-1}, i - 1) = \phi$, a contradiction. If $i - 1 > 2n - 1$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction. Therefore, $D_{ct}(TL_n, i) \neq \phi$.

(v) Since, $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$, by Lemma 2.2 (iv), we have, $n - 2 \leq i - 1 \leq 2n - 1$. Also, since, $D_{ct}(TL_{n-1}, i - 1) = \phi$, by Lemma 2.2 (iii), we have, $i - 1 < n - 2$ or $i - 1 > 2n - 2$. If $i - 1 < n - 2$, then $D_{ct}(TL_n - \{2n\}, i - 1) = \phi$, a contradiction. Therefore, $i - 1 > 2n - 2$. Therefore, $i - 1 \geq 2n - 1$. Also, $i - 1 \leq 2n - 1$. Therefore, $i - 1 = 2n - 1$. Therefore, $i = 2n$. Hence, $D_{ct}(TL_n, i) \neq \phi$.

Lemma 2.4

Suppose that $D_{ct}(TL_n, i) \neq \phi$, then for every $n \in \mathbb{N}$,

- (i) $D_{ct}(TL_n - \{2n\}, i - 1) \neq \phi$ and $D_{ct}(TL_{n-1}, i - 1) = \phi$

if and only if $i = 2n$.

- (ii) $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$ and
 $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) = \phi$, if and only if $i = 2n-1$.
- (iii) $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$,
 $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) \neq \phi$ and
 $D_{ct}(TL_{n-2}, i-1) = \phi$ if and only if $i = 2n-2$.

Proof

Assume that, $D_{ct}(TL_n, i) \neq \phi$.

Therefore, $n-1 \leq i \leq 2n$.

- (i) $(\Rightarrow)$ since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, by Lemma 2.2 (iv), we have,
 $n-2 \leq i-1 \leq 2n-1$.
 Therefore, $n-1 \leq i \leq 2n$.
 Also, since, $D_{ct}(TL_{n-1}, i-1) = \phi$, by Lemma 2.2 (iii), we have,
 $i-1 < n-2$ or $i-1 > 2n-2$.
 If $i-1 < n-2$, then $i < n-1$ which implies
 $D_{ct}(TL_n, i) = \phi$, a contradiction.
 Therefore, $i-1 > 2n-2$.
 Therefore, $i > 2n-1$.
 Therefore, $i \geq 2n$.
 Together we have, $i = 2n$.
 $(\Leftarrow)$ follows from Lemma 2.2 (iii) & (iv).
- (ii) $(\Rightarrow)$ Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, and $D_{ct}(TL_{n-1}, i-1) \neq \phi$,
 by Lemma 2.2 (iii) & (iv), we have,
 $n-2 \leq i-1 \leq 2n-1$ and
 $n-2 \leq i-1 \leq 2n-2$.
 Therefore, $n-2 \leq i-1 \leq 2n-2$.
 Therefore, $n-1 \leq i \leq 2n-1$.
 Also, since, $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) = \phi$,
 by Lemma 2.2 (iv), we have, $i-1 < n-3$ or
 $i-1 > 2n-3$.

Therefore, $i < n-2$ or $i > 2n-2$.

If $i < n-2$, the $i < n-1$ holds, which implies

$D_{ct}(TL_n, i) = \phi$, a contradiction.

Therefore, $i > 2n-2$.

Therefore, $i \geq 2n-1$.

Together we have, $i = 2n-1$.

$(\Leftarrow)$ follows from Lemma 2.2 (iii) & (iv).

- (iii) $(\Rightarrow)$ Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$, $D_{ct}(TL_{n-1}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1} - \{2n-2\}, i-1) \neq \phi$, by Lemma 2.2 (iii) & (iv), we have,

$$n-2 \leq i-1 \leq 2n-1,$$

$$n-2 \leq i-1 \leq 2n-2 \text{ and}$$

$$n-3 \leq i-1 \leq 2n-3.$$

Therefore, $n-2 \leq i-1 \leq 2n-3$.

Therefore, $n-1 \leq i \leq 2n-2$.

Also, since, $D_{ct}(TL_{n-2}, i-1) = \phi$, by Lemma 2.2 (iii), we have, $i-1 < n-3$, or $i-1 > 2n-4$.

If $i-1 < n-3$, then $i < n-2$.

Therefore, $i < n-1$ holds, which implies $D_{ct}(TL_n, i) = \phi$, a contradiction.

Therefore, $i-1 > 2n-4$.

Therefore, $i > 2n-3$.

Therefore, $i \geq 2n-2$.

Together we have $i = 2n-2$.

$(\Leftarrow)$ follows from Lemma 2.2 (iii) & (iv).

Theorem 2.5

For every $n \geq 4$,

- (i) If $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) = \phi$,
 then $D_{ct}(TL_n, i) = \{X \cup \{2n\} / X \in D_{ct}(TL_n - \{2n\}, i-1)\}$.
- (ii) If $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) \neq \phi$,
 then
 $D_{ct}(TL_n, i) = \{X_1 \cup \{2n-1\}, \text{ if } 2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}, \text{ if } 2n-2 \text{ or } 2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_2 \cup \{2n-2\}, \text{ if } 2n-4 \text{ or } 2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}, \text{ if } 2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.

Proof

- (i) Since, $D_{ct}(TL_n - \{2n\}, i-1) \neq \phi$ and $D_{ct}(TL_{n-1}, i-1) = \phi$,
 by Theorem 2.4 (i), $i = 2n$.

Therefore, $D_{ct}(TL_n, i) = D_{ct}(TL_n, 2n) = \{[2n]\}$ and

$D_{ct}(TL_n - \{2n\}, i-1) = D_{ct}(TL_n - \{2n\}, 2n-1) = \{[2n-1]\}$,
 we have the result.

- (ii) Let $Y_1 = \{X_1 \cup \{2n-1\}, \text{ if } 2n-3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\} \cup \{X_1 \cup \{2n\}, \text{ if } 2n-2 \text{ or } 2n-1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i-1)\}$
 and $Y_2 = \{X_2 \cup \{2n-2\}, \text{ if } 2n-4 \text{ or } 2n-3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\} \cup \{X_2 \cup \{2n-1\}, \text{ if } 2n-2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i-1)\}$.

Obviously, $Y_1 \cup Y_2 \subseteq D_{ct}(TL_n, i)$ (1)

Now, let $Y \in D_{ct}(TL_n, i)$.

If $2n \in Y$, then atleast one of the vertices labeled $2n-2$ or $2n-1$ is in Y . In either cases, $Y = \{X_1 \cup \{2n\}\}$ for some $X_1 \in D_{ct}(TL_n - \{2n\}, i-1)$.

Therefore, $Y \in Y_1$.

Suppose that, $2n-1 \in Y$, $2n \notin Y$, then atleast one of the vertices labeled $2n-3$ or $2n-2$ is in Y .

If $2n-3 \in Y$, then $Y = \{X_1 \cup \{2n-1\}\}$ for some

$X_1 \in D_{ct}(TL_n - \{2n\}, i-1)$.

If $2n-2 \in Y$, then $Y = \{X_2 \cup \{2n-1\}\}$ for some

$X_2 \in D_{ct}(TL_{n-1}, i-1)$.

Therefore, $Y \in Y_1$ or $Y \in Y_2$.

Now, Suppose that,

$2n - 2 \in Y$, $2n - 1 \notin Y$, $2n \notin Y$, then atleast one of the vertices labeled $2n - 4$ or $2n - 3$ is in Y .

In either cases, $Y = \{X_2 \cup \{2n - 2\}\}$ for some

$X_2 \in D_{ct}(TL_{n-1}, i-1)$.

Therefore, $Y \in Y_2$.

Hence, $D_{ct}(TL_n, i) \subseteq Y_1 \cup Y_2$ (2)

From (1) and (2), we have,

$D_{ct}(TL_n, i) = \{X_1 \cup \{2n - 1\}, \text{ if } 2n - 3 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i - 1)\} \cup \{X_1 \cup \{2n\}, \text{ if } 2n - 2 \text{ or } 2n - 1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i - 1)\} \cup \{X_2 \cup \{2n - 2\}, \text{ if } 2n - 4 \text{ or } 2n - 3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i - 1)\} \cup \{X_2 \cup \{2n - 1\}, \text{ if } 2n - 2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i - 1)\}$.

Theorem 2.6

If $D_{ct}(TL_n, i)$ be the family of connected total dominating sets of TL_n with cardinality i , where $i \geq n - 1$, then $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i - 1) + d_{ct}(TL_{n-1}, i - 1)$.

Proof

We consider the two cases given in Theorem 2.5.

By Theorem 2.5 (i), we have,

$D_{ct}(TL_n, i) = \{X \cup \{2n\} / X \in D_{ct}(TL_n - \{2n\}, i - 1)\}$.

Since, $D_{ct}(TL_{n-1}, i - 1) = \emptyset$, we have,

$d_{ct}(TL_{n-1}, i - 1) = 0$.

Therefore, $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i - 1)$.

By Theorem 2.5 (ii), we have,

$D_{ct}(TL_n, i) = \{X_1 \cup \{2n - 1\}, \text{ if } 2n - 3 \in X_1 / X_1 \in$

$D_{ct}(TL_n - \{2n\}, i - 1)\} \cup \{X_1 \cup \{2n\}, \text{ if } 2n - 2 \text{ or }$

$2n - 1 \in X_1 / X_1 \in D_{ct}(TL_n - \{2n\}, i - 1)\} \cup \{X_2 \cup \{2n - 2\},$

$\text{if } 2n - 4 \text{ or } 2n - 3 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i - 1)\} \cup$

$\{X_2 \cup \{2n - 1\}, \text{ if } 2n - 2 \in X_2 / X_2 \in D_{ct}(TL_{n-1}, i - 1)\}$.

Therefore,

$d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i - 1) + d_{ct}(TL_{n-1}, i - 1)$.

3. Connected total domination polynomials of triangular ladders

Definition 3.1

Let $D_{ct}(TL_n, i)$ be the family of connected total dominating sets of TL_n with cardinality i and let $d_{ct}(TL_n, i) = |D_{ct}(TL_n, i)|$.

Then the connected total domination Polynomial

$D_{ct}(TL_n, x)$ of TL_n is defined as,

$D_{ct}(TL_n, x) = \sum_{i=\gamma_{ct}(TL_n)}^{2n} d_{ct}(TL_n, i) x^i$.

Theorem 3.2

For every $n \geq 4$,

$D_{ct}(TL_n, x) = x[D_{ct}(TL_n - \{2n\}, x) + D_{ct}(TL_{n-1}, x)]$, with initial values,

$D_{ct}(TL_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(TL_2, x) = 5x^2 + 4x^3 + x^4$.

$D_{ct}(TL_3 - \{6\}, x) = 5x^2 + 8x^3 + 5x^4 + x^5$.

$D_{ct}(TL_3, x) = 3x^2 + 10x^3 + 12x^4 + 6x^5 + x^6$.

$D_{ct}(TL_4 - \{8\}, x) = x^2 + 8x^3 + 18x^4 + 17x^5 + 7x^6 + x^7$.

Proof

We have, $d_{ct}(TL_n, i) = d_{ct}(TL_n - \{2n\}, i - 1) + d_{ct}(TL_{n-1}, i - 1)$.

Therefore, $d_{ct}(TL_n, i) x^i = d_{ct}(TL_n - \{2n\}, i - 1) x^i + d_{ct}(TL_{n-1}, i - 1) x^i$.

$\sum d_{ct}(TL_n, i) x^i = \sum d_{ct}(TL_n - \{2n\}, i - 1) x^i + \sum d_{ct}(TL_{n-1}, i - 1) x^i$.

$\sum d_{ct}(TL_n, i) x^i = x \sum d_{ct}(TL_n - \{2n\}, i - 1) x^{i-1} + x \sum d_{ct}(TL_{n-1}, i - 1) x^{i-1}$.

$D_{ct}(TL_n, x) = x D_{ct}(TL_n - \{2n\}, x) + x D_{ct}(TL_{n-1}, x)$.

Therefore, $D_{ct}(TL_n, x) = x[D_{ct}(TL_n - \{2n\}, x) + D_{ct}(TL_{n-1}, x)]$ with initial values,

$D_{ct}(TL_2 - \{4\}, x) = 3x^2 + x^3$.

$D_{ct}(TL_2, x) = 5x^2 + 4x^3 + x^4$.

$D_{ct}(TL_3 - \{6\}, x) = 5x^2 + 8x^3 + 5x^4 + x^5$.

$D_{ct}(TL_3, x) = 3x^2 + 10x^3 + 12x^4 + 6x^5 + x^6$.

$D_{ct}(TL_4 - \{8\}, x) = x^2 + 8x^3 + 18x^4 + 17x^5 + 7x^6 + x^7$.

Example 3.3

$D_{ct}(TL_4, x) = 4x^3 + 18x^4 + 30x^5 + 23x^6 + 8x^7 + x^8$.

$D_{ct}(TL_5 - \{10\}, x) = x^3 + 12x^4 + 36x^5 + 47x^6 + 30x^7 + 9x^8 + x^9$.

By Theorem 3.2, we have,

$D_{ct}(TL_5, x) = x [4x^3 + 18x^4 + 30x^5 + 23x^6 + 8x^7 + x^8 + x^3 + 12x^4 + 36x^5 + 47x^6 + 30x^7 + 9x^8 + x^9]$
 $= 5x^4 + 30x^5 + 66x^6 + 70x^7 + 38x^8 + 10x^9 + x^{10}$.

We obtain $d_{ct}(TL_n, i)$ and $d_{ct}(TL_n - \{2n\}, i)$ for $2 \leq n \leq 9$ as shown in Table 1.

Table 1

$\begin{matrix} i \\ n \end{matrix}$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$TL_2-\{4\}$	3	1															
TL_2	5	4	1														
$TL_3-\{6\}$	5	8	5	1													
TL_3	3	10	12	6	1												
$TL_4-\{8\}$	1	8	18	17	7	1											
TL_4	0	4	18	30	23	8	1										
$TL_5-\{10\}$	0	1	12	36	47	30	9	1									
TL_5	0	0	5	30	66	70	38	10	1								
$TL_6-\{12\}$	0	0	1	17	66	113	100	47	11	1							
TL_6	0	0	0	6	47	132	183	138	57	12	1						
$TL_7-\{14\}$	0	0	0	1	23	113	245	283	185	68	13	1					
TL_7	0	0	0	0	7	70	245	428	421	242	80	14	1				
$TL_8-\{16\}$	0	0	0	0	1	30	183	490	711	606	310	93	15	1			
TL_8	0	0	0	0	0	8	37	428	918	1132	848	390	107	16	1		
$TL_9-\{18\}$	0	0	0	0	0	1	38	220	918	1629	1738	1158	483	122	17	1	
TL_9	0	0	0	0	0	0	9	75	648	1836	2761	2586	1548	590	138	18	1

In the following Theorem we obtain some properties of $d_{ct}(TL_n, i)$.

Theorem 3.4

The following properties hold for the coefficients of $D_{ct}(TL_n, x)$ for all n .

- (i) $d_{ct}(TL_n, 2n) = 1$, for every $n \geq 2$.
- (ii) $d_{ct}(TL_n, 2n-1) = 2n$, for every $n \geq 2$.
- (iii) $d_{ct}(TL_n, 2n-2) = 2n^2 - 3n + 3$, for every $n \geq 2$.
- (iv) $d_{ct}(TL_n, n-1) = n$, for every $n \geq 3$.

Proof

Proof is obvious.

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